Does school desegregation promote diverse interactions? An equilibrium model of segregation within schools*

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Abstract

This paper studies racial segregation in schools using data on student friendships from Add Health. I estimate a structural equilibrium model of friendship formation among students, with preferences that allow for both homophily (a bias for similar people) and heterophily (a bias for different people) on different characteristics. Preferences also depend on link externalities, such as having common friends or reciprocated links. I find that students tend to interact with similar people. Homophily goes beyond direct links: students also prefer a racially homogeneous set of indirect friends. However, I find heterophily in parental income levels and for the group of hispanic students. I simulate several re-allocation programs, showing that policies that transport minorities to other schools have nonlinear effects on within-school segregation and other network features such as clustering and centrality. In some instances, these interventions increase segregation within schools. Policies that increase racial diversity by re-allocation of student according to their parental income have less impact on racial segregation with schools.

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1 Introduction

Social networks are important determinants of individuals' socioeconomic performance. An increasing amount of evidence shows that the number and composition of social ties affects employment prospects, school performance, risky behavior, adoption of new technologies, diffusion of information and health outcomes (Topa, 2001; Laschever, 2009; Cooley, 2010; De Giorgi et al., 2010; Nakajima, 2007; Bandiera and Rasul, 2006; Conley and Udry, 2010; Golub and Jackson, 2011; Acemoglu et al., 2011; Calvo-Armengol et al., 2009). An empirical challenge in estimating such network effects is the endogeneity of social relationship: individuals choose their peers and friends according to their socioeconomic characteristics and their relationships. As a consequence, in socially generated networks the agents are likely to interact with similar individuals (homophily), segregating along socioeconomic attributes (Currarini et al., 2009; Boucher, 2015; De Marti and Zenou, 2009).

Empirical estimates of peer effects suggest that segregation may have an effect on school outcomes of minorities (Echenique and Fryer, 2007; Cutler and Glaeser, 1997; Ananat, 2011; Angrist and Lang, 2004; Echenique et al., 2006). Desegregation programs are an attempt to re-engineer this process, by re-allocation of minority students to predominantly white/caucasian schools. The underlying premise about desegregation programs is that by exposing students of different racial backgrounds to each other, they will necessarily interact more. In practice, these programs face a Lucas critique: if desegregation plans are designed without taking into account the equilibrium choice of peers and the effects on outcomes, they may backfire (Carrell et al., 2013; Badev, 2013). In an experimental study that re-assigned students to peer groups to maximize educational outcomes based on peer effects estimates, Carrell et al. (2013) show that after being assigned to another group, students modify the way they interact within the newly assigned group, decreasing the expected effect of the intervention. Recent structural models suggest that ignoring the equilibrium network formation leads to underestimating peer effects (Badev, 2013; Hsieh and Lee, 2012).

This paper contributes to this debate by estimating an equilibrium model of network formation in schools and measuring the extent to which re-allocations of students among schools lead to changes in segregation patterns and interactions, *even in the absence of peer effects*. While from a policy perspective it is not clear whether we should focus on segregation across schools or within schools to improve outcomes, my simulations show that there is a relationship between the demographic composition of the school and the way students interact in groups.[1] These simulations can thus be interpreted as measuring a first-order effect of such re-allocation policies. In the model, students are heterogeneous on several (observed) dimensions and decide whether to form links to other students based on their preferences. The preferences are flexible and allow for both homophily (a bias for similar people) and heterophily (a bias for different people) on different characteristics (Mele, 2017*a*; Boucher, 2015; DePaula et al., forthcoming). For example, there may be homophily by race and heterophily by age. Preferences also depend on link externalities, such as having common friends or reciprocated links.

The link formation is sequential and we focus on stationary equilibria. The reason is that data usually come from a single snapshot of the network, therefore the dynamics of the model is harder to identify and estimate. The stationary equilibrium is characterized by the probability of observ-

[1] Most of the literature focuses on the effects of segregation across schools (Clotfelter, 2004; Lutz, 2011; Angrist and Lang, 2004), there are recent works that analyze the effect of segregation within schools (Echenique and Fryer, 2007; Echenique et al., 2006; Badev, 2013; Mele, 2017*b*) on outcomes.

ing a particular network in the long run. This distribution has peaks at the networks that are Nash equilibria. Assuming that the network we observe in the data is drawn from the stationary equilibrium of the model, this distribution can be interpreted as the likelihood of observing a network in the long run.

Compared to a standard logit model of link prediction, the model used in this paper incorporates externalities and equilibrium effects that change the incentives of a player to form or delete links. Indeed if the equilibrium effects are absent the only relevant determinants of a link are the demographics of the players. An individual will form a link to another student if the direct utility derived from the demographic characteristics is sufficiently high (net of idiosynchratic shocks). However, this mechanism does not take into account that when a student forms a new link, she is also creating an additional indirect link for someone else. While this indirect connection may not bear the full extent of benefits of a direct link, it may increase or decrease the payoffs. This equilibrium mechanism generates additional incentives (positive or negative) to form links, and may as well depend on demographics and compositions of indirect ties. We focus on the equilibrium of such a process of link formation, that is networks that are stable in a well defined sense (game theoretical equilibrium).

Using this theoretical framework, I estimate preferences for link formation in school friendship networks, using data from the *National Longitudinal Study of Adolescent Health* (Add Health). This unique dataset contains detailed information on friendship networks of students enrolled in a representative sample of US schools. The final sample includes 16 high schools from the *saturated sample* with a total of 3604 students and more than 3.5 million pairs.[2] I find that race, gender, parental income, attractiveness and grade are important determinants of network formation in schools. There is overwhelming evidence of homophily: students tend to interact and form social ties with similar people, controlling for the structure of their network. Furthermore, I find that homophily effects extend well beyond direct links: for example, students also prefer an homogeneous racial composition of friends of friends. I also find evidence of heterophily in parental income levels, suggesting that students tend to mix by socioeconomic status. Interestingly, hispanics prefer a diverse group of friends, as well as a diverse group of indirect friends. Therefore, the creation of a new link with a student of the same race, will change the incentive of other students to form links within the same racial group. If these incentives are positive (homophily) this effect will lead to increased segregation levels and clustering of the network; viceversa, if there is heterophily, this leads to a decrease in segregation and clustering.

I use the estimated model to predict how a change in the composition of the student population affects the structure of the network.[3] Because the structure of the network is correlated with individual and aggregate outcomes, this effect is of first-order importance (Carrell et al., 2013; Echenique and Fryer, 2007; Badev, 2013; Calvo-Armengol et al., 2009). First, I simulate a desegregation program by swapping students in two homogeneous schools that are (almost) completely white and black, respectively. Interestingly, the results depend on which measure of network seg-

[2] I use only the schools from the *saturated sample*. The sampling scheme of Add Health involved in-school interviews for all 90,118 students. A subsample of 20745 students was also interviewed at home, to collect detailed individual information. The saturated sample contains schools for which both interviews were administered to *each* student enrolled. Therefore this sample does not contain any missing information about individual controls. This is not the case for most schools in Add Health.

[3] Alternatively, the model could be used as a guide for the design of randomized experiments that modify students assignments.

regation we use. The most intuitive measure of segregation, the Freeman Segregation Index (FSI), measures the difference between the fraction of mixed-group friendships and their expected value under no homophily. The conclusion of this experiment is that trying to make schools equally diverse by equalizing their racial share may not decrease segregation within schools, according to FSI. Indeed, segregation is lower in presence of a relatively small minority, that is not able to self-segregate. While the pattern of segregation as a function of the fraction of blacks in the school is similar, there are differences in the magnitude across schools, due to the difference on how whites and blacks respond to changes in the fraction of their own groups in the school. Income segregation slightly decreases in the (originally) white school, while it increases in the (originally) black school. These conclusions change if we measure segregation using the Spectral Segregation Index (SSI) of Echenique and Fryer (2007). SSI measures segregation of a group in a way similar to how the page-rank algorithm computes scores for webpages. That is, the segregation of a student is higher, the higher the segregation of the students she is linked to. We use the mean SSI to measure segregation in the school, and our results show that an increase in the fraction of blacks in the school will always increase segregation. Therefore, a policy that makes schools equally diverse will be a good compromise in terms of racial and income segregation.

Second, in another policy experiment, I take some students whose parent incomes are above median in a homogeneously white school and swap them with students with incomes below median in a homogeneously black school. This policy increases income segregation across schools, while also increasing racial diversity within schools.[4] There is almost no effect on the level of racial segregation within schools, according to the FSI. On the other hand, the effect on income segregation is different across the two schools. The school that is homogeneously white becomes poorer and sees an increase in income segregation. The other school becomes richer and sees almost no change in income segregation. According to the SSI, racial and income segregation will increase in the white school and decrease in the black school. Finally while in the black school the most popular students are black, in the white school this policy increases the probability that the most central (and popular kids) will be blacks.

In summary, I show that even absent peer effects or other outcome-related motives for link formation, a reassignment of students will result in a new equilibrium set of links. The presence of link externalities in the payoffs may amplify this mechanism, resulting in more or less segregation by socioeconomic characteristics, a point of practical importance when designing desegregation programs.

This paper contributes to the empirical network literature by estimating an equilibrium model of homophily and segregation (Jackson (2008), DePaula (forthcoming), Graham (2017), Chandrasekhar (2016)),[5] whose results are consistent with recent evidence on homophily in networks (Currarini et al., 2009; Boucher, 2015; Boucher and Mourifie, forthcoming; Mele, 2017*b*). I also find homophily for indirect connections. Models of network formation that exclude link externalities in the payoffs are not able to capture this feature (see for example Graham (2017), Dzemski (2017)), attributing all the homophily to direct links or unobserved characteristics.

The rich dataset used in my estimation partially solves the identification issues highlighted in Mele (2017*a*). Since I estimate the model using multiple independent school networks, the identi-

[4]This simulation is motivated by some evidence that the disparity in average school poverty rates between white and black students schools is a strong predictor of racial school achievement gaps (Reardon, 2016).

[5]For additional structural models of network formation see Menzel (2015), Sheng (2012), DePaula et al. (forthcoming), Leung (2014*b*), Leung (2014*a*).

fication of the structural parameters relies on both variation of network features across schools and within schools (Lehman, 1983; Wainwright and Jordan, 2008).[6]

The paper also contributes to the literature on racial segregation in schools. This body of work focused on the effects of residential and school segregation on minority socioeconomic outcomes (Cutler and Glaeser (1997), Echenique and Fryer (2007), La Ferrara and Mele (2011), Ananat (2011)). Other work has considered the effect of school segregation on educational attainment (Angrist and Lang (2004)). Most studies analyze segregation among schools in a district, but few authors have used more detailed data at the school level to understand the patterns of racial segregation within schools (Echenique and Fryer (2007), Echenique et al. (2006), Mele (2017*b*), Boucher (2015), Badev (2013)). While it is not clear whether within- or between-schools segregation is more important in determining outcomes, my approach in this paper provides a structural interpretation of the segregation levels within-schools and takes between-schools segregation as given. In the policy experiments I modify between-schools segregation by simulating reallocations of the students, focusing on the equilibrium implications within schools.

This model also provides insights about possible mechanisms behind the moderate increase in racial segregation after the end of court-ordered desegregation programs (Lutz, 2011) and modest effects of desegregation program on educational outcomes (Angrist and Lang, 2004).

The rest of the paper is organized as follows. Section 2 briefly describes the theoretical model and provide the intuition about the equilibrium effects. Section 3 develops the estimation strateg, and provides an overview of the data and identification. Section 4 report the posterior estimates and the policy experiments. Some of the computational details are provided in Appendix.

2 A Model of Network Formation

In this section, I briefly present the setup of Mele (2017*a*)'s model and the equilibrium likelihood. The proofs and more theoretical results are all contained in that paper. Time is discrete and there are n players in the network. Each player is characterized by a vector of observable covariates X_i, that may contain information about race, gender, wealth, age, location, etc. The matrix $X = \{X_1, X_2, ..., X_n\}$ contains these vectors for all the players, stacked by column.

The social network of friendship nominations is represented by a $n \times n$ adjacency matrix g, with entries $g_{ij} = 1$ if individual i nominates individual j as a friend, and $g_{ij} = 0$ otherwise. There are no self-loops, $g_{ii} = 0$, for any i, and the network is *directed*: the existence of a link from i to j does not imply the existence of the link from j to i. This modeling choice reflects the structure of the Add Health data, where friendship nominations are not necessarily mutual. Some authors refer to this data as *perceived* networks.[7] Let the *realization* of the network at time t be denoted as g^t and the *realization* of the link between i and j at time t be g_{ij}^t. The network including all the current links but g_{ij}^t, i.e. $g^t \backslash g_{ij}^t$, is denoted as g_{-ij}^t; while g_{-i}^t denotes the network matrix excluding the i-th row (i.e. all the links of player i).

The network formation process follows a stochastic best-response dynamics as in Blume (1993). At the beginning of each period a player i is randomly selected from the population, and he meets individual j, according to a meeting probability $\rho(ij|g^{t-1}, X)$. Notice that ρ may depend on the previous period network and the observable characteristics. For example, people that have many

[6]Similar identification with multiple network is exploited in Nakajima (2007), Badev (2013), Sheng (2012).

[7]See Wasserman and Faust (1994) for references.

friends in common may meet with higher probability than people without common friends. Or students with similar demographics have higher probability of interaction than students with different backgrounds. An implicit assumption of the model is that the player can observe the entire network and the covariates of all the agents, before making their choice about linking.

Upon meeting agent j, player i decides whether to update his link g_{ij}. The preferences of i are defined over networks and covariates. The utility of player i from network g and covariates X is given by

$$U_i(g, X) = \underbrace{\sum_{j=1}^{n} g_{ij} u_{ij}}_{\text{direct friends}} + \underbrace{\sum_{j=1}^{n} g_{ij} g_{ji} m_{ij}}_{\text{mutual friends}} + \underbrace{\sum_{j=1}^{n} g_{ij} \sum_{\substack{k=1 \\ k \neq i,j}}^{n} g_{jk} v_{ik}}_{\text{friends of friends}} + \underbrace{\sum_{j=1}^{n} g_{ij} \sum_{\substack{k=1 \\ k \neq i,j}}^{n} g_{ki} w_{kj}}_{\text{popularity}} \tag{1}$$

where $u_{ij} \equiv u(X_i, X_j)$, $m_{ij} \equiv m(X_i, X_j)$, $v_{ij} \equiv v(X_i, X_j)$ and $w_{ij} \equiv w(X_i, X_j)$ are (bounded) real-valued functions of the attributes. The utility of the network is the sum of the net benefits received from each link. The total benefit from an *additional link* has four components.

When a player creates a link to another individual, he receives a *direct* net benefit u_{ij}. The direct utility includes both costs and benefits and it may possibly be negative: when only homophily enters payoffs of direct links, the net utility u_{ij} is positive if i and j belong to the same group, while it is negative when they are of different types. In many models this component is parameterized as $u_{ij} = b_{ij} - c_{ij}$, where b_{ij} indicates the (gross) benefit and c_{ij} the cost of forming the additional link g_{ij}. I use the notation u_{ij}, since it does not require assumptions on the cost function.

The players receive additional utility m_{ij} if the link is mutual; friendship is valued differently if the other agent reciprocates. An agent may perceive another individual as a friend, but that person may not perceive the relationship in the same way.

The players value the composition of friends of friends. When i is deciding whether to befriend j, she observes j's friends and their socioeconomic characteristics. Each of j's friend provides additional utility $v(X_i, X_k)$ to i. In this model, an agent who has the opportunity to form an additional link, values a white student with three Hispanic friends as a different good than a white student with two white friends and one African American friend.[8] In other words, individuals value both *exogenous* heterogeneity and *endogenous* heterogeneity: the former is determined by the socioeconomic characteristics of the agents, while the latter arises endogenously with the process of network formation. I assume that only friends of friends are valuable and they are perfect substitutes: individuals do not receive utility from two-links-away friends.

The fourth component corresponds to a *popularity effect*. When an agent forms a link, he/she automatically creates an indirect link for other agents that are connect to him/her, thus generating externalities. This makes him/her more or less appealing to his friends, thus impacting his/her "popularity".

Conditional on the meeting $m^t = ij$, player i updates the link g_{ij} to maximize his current utility, taking the existing network g^t_{-ij} as given. I assume that the agents do not take into account the effect of their linking strategy on the future evolution of the network. The players have *complete information*, since they can observe the entire network and the individual attributes of all

[8] A similar assumption is used in De Marti and Zenou (2009) where the agents' cost of linking depend on the racial composition of friends of friends. Their model is an extension of the connection model of Jackson and Wolinsky (1996), and the links are formed with mutual consent. The corresponding network is undirected.

agents. Before updating his link to j, individual i receives an idiosyncratic shock $\varepsilon \sim F(\varepsilon)$ to his preferences that the econometrician cannot observe. This shock models unobservables that could influence the utility of an additional link, e.g. mood, gossips, fights, etc. Player i links agent j at time t if and only if it is a best response to the current network configuration, i.e. $g_{ij}^t = 1$ if and only if

$$U_i\left(g_{ij}^t = 1, g_{-ij}^{t-1}, X\right) + \varepsilon_{1t} \geq U_i\left(g_{ij}^t = 0, g_{-ij}^{t-1}, X\right) + \varepsilon_{0t}. \tag{2}$$

I assume that when the equality holds, the agent plays the status quo.[9]

The model satisfies the following assumptions:

Assumption 1. (Preferences) The payoffs are such that $m(X_i, X_j) = m(X_j, X_i)$ and $w(X_k, X_j) = v(X_k, X_j)$ for all players i, j, k.

Assumption 2. (Meetings) Any meeting is possible, i.e., $\rho(ij|g^{t-1}, X) = \rho(ij|g_{-ij}^t, X) > 0$ for any pair of players i, j.

Assumption 3. (Shocks) Before deciding whether to update a link, players receive a stochastic shock that follows a Type I extreme value distribution, i.i.d. among links and across time.

The first assumption about symmetry of m_{ij} is needed for identification: two individuals with the same exogenous characteristics $X_i = X_j$ (say two males, whites, enrolled in eleventh grade) who form a mutual link receive the same u_{ij} and m_{ij}, but they may have different utilities from that additional link because of the composition of their friends of friends and their popularity. Therefore, this part of the assumption helps in identifying the utility from indirect links and popularity.

When i forms a link to j, i creates an externality for all k's who have linked her: any such k now has an additional indirect friend, i.e. j, who agent k values by an amount $v(X_k, X_j)$. When $w(X_k, X_j) = v(X_k, X_j)$, an individual i values his popularity effect as much as k values the indirect link to j, i.e., i internalizes the externality he creates.[10]

The second assumption on the meeting process gurantees that any pair of agents can meet. The main implication is that any equilibrium network can be reached with positive probability. For example, a discrete uniform distribution satisfies this assumption.

Finally the third assumption allows the Markov chain to escape from the nash networks, eliminating absorbing states and making the model ergodic.

As a consequence of these assumptions, the network formation process is a potential game, where all the incentives of the players can be summarized by an aggregate function of the network Q.

$$Q(g, X) = \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}u_{ij} + \sum_{i=1}^{n}\sum_{j>i}^{n} g_{ij}g_{ji}m_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\sum_{\substack{k=1 \\ k\neq i,j}}^{n} g_{ij}g_{jk}v_{ik}, \tag{3}$$

[9] This assumption does not affect the main result and is relevant only when the distribution of the preference shocks is discrete.

[10] This restriction of the preferences guarantees the model's coherency in the sense of Tamer (2003). In simple words, this part of the assumption guarantees that the system of conditional linking probabilities implied by the model generates a proper joint distribution of the network matrix. Similar restrictions are also encountered in spatial econometrics models (Besag, 1974) and in the literature on qualitative response models (Heckman, 1978; Amemiya, 1981)

The potential is such that, for any player i and any link g_{ij} we have

$$Q\left(g_{ij}, g_{-ij}, X\right) - Q\left(1 - g_{ij}, g_{-ij}, X\right) = U_i\left(g_{ij}, g_{-ij}, X\right) - U_i\left(1 - g_{ij}, g_{-ij}, X\right)$$

Consider two networks, $g = (g_{ij}, g_{-ij})$ and $g' = (1 - g_{ij}, g_{-ij})$, that differ only with respect to one link, g_{ij}, chosen by individual i: the difference in utility that agent i receives from the two networks, $U_i(g, X) - U_i(g', X)$, is exactly equal to the difference of the *potential* function evaluated at the two networks, $Q(g, X) - Q(g', X)$. That is, the potential is an aggregate function that summarizes both the state of the network and the deterministic incentives of the players in each state.

The model generates a Markov Chain of networks that converges to a uniques stationary distribution π

$$\pi(g, X) = \frac{\exp\left[Q(g, X)\right]}{\sum_{\omega \in \mathcal{G}} \exp\left[Q(\omega, X)\right]}, \tag{4}$$

where $\mathcal{G}$ is the set of all networks with n nodes. In the long-run the systems spends more time in network states that have high potential. It can be shown that these networks correspond to Nash equilibria of a model without any shock to the preferences (Mele (2017*a*), Jackson and Watts (2001), Monderer and Shapley (1996)). The function

$$c(\mathcal{G}, X, \theta) = \sum_{\omega \in \mathcal{G}} \exp\left[Q(\omega, X, \theta)\right]. \tag{5}$$

is a normalizing constant, that guarantees that (4) is a proper probability. This is the sum of exponential potential functions over the set $\mathcal{G}$ of all networks with n nodes.

2.1 The role of indirect payoffs in equilibrium

To develop the intuition about the role of indirect payoffs in the determination of the equilibrium, let's consider a simplified version of this model with only two types of students (e.g. boys and girls, whites and nonwhites, etc.). The type is a binary variable $x_i \in \{0, 1\}$; let the indicator function $\mathbf{1}_{ij} = 1$ if $x_i = x_j$ and $\mathbf{1}_{ij} = 0$ otherwise. Let the direct payoff be $u_{ij} = \alpha + \beta\mathbf{1}_{ij}$, the mutual utility $m_{ij} = 0$ and the indirect utility $v_{ij} = \gamma\mathbf{1}_{ij}$ for any $i, j = 1, ..., n$. I assume $\alpha < 0$ for the rest of this section, and interpret α as cost of forming a link; so for every link g_{ij}, student i pays a cost α; if j's type is the same as i, that is $x_i = x_j$, student i receives an additional direct payoff β; finally, student i receives a payoff γ for each indirect friend of j whose type is the same as i's. Therefore the utility of player i is

$$U_i(g, X) = \alpha \sum_{j=1}^{n} g_{ij} + \beta \sum_{j=1}^{n} g_{ij}\mathbf{1}_{ij} + \gamma \sum_{j=1}^{n} g_{ij} \sum_{\substack{k=1 \\ k \neq i,j}}^{n} \left(g_{jk}\mathbf{1}_{ik} + g_{ki}\mathbf{1}_{kj}\right) \tag{6}$$

There is same-type bias (homophily) in preferences when $\beta > 0$ and indirect same-type bias when $\gamma > 0$. If $\beta < 0$ we have bias for other types, and $\gamma < 0$ implies indirect bias for other types. Depending on the magnitude of β and γ we can observe homophily or heterophily in aggregate. The role of α is to drive the density of the network. If $\alpha < 0$ and large enough we will have relatively sparse networks, while $\alpha > 0$ will necessarily generate dense networks. More generally the density of the network will depend on the relative magnitude of α, β and γ.

No indirect effects. To understand the equilibrium feedbacks implied by the model let's first shut down all payoffs from indirect links, setting $\gamma = 0$. Conditional on i meeting j, a link is formed with probability $p_{ij}^s = \exp(\alpha + \beta)$ if $x_i = x_j$; and $p_{ij}^d \propto \exp(\alpha)$ if $x_i \neq x_j$. Therefore we expect the model to converge to a network with $n(n-1)p_{ij}^s$ links among people of the same type and $n(n-1)p_{ij}^d$ links among people of different type. Because there is no indirect payoffs, the equilibrium network will be welfare maximizing. The level of homophily depends on the parameter β: if $\beta > 0$ and large, we will observe most links between people of same type; viceversa, if $\beta < 0$ most links will be between people of different type.

Positive indirect payoffs. If $\gamma > 0$ and $\beta > 0$ the homophily effect will be amplified by the indirect payoff. Indeed, students will tend to form links to same type people with a homogeneous set of friends of the same type. Compared with the case $\gamma = 0$ above, this case will lead to a network with more same-type links and more clustering, with the creation of very dense communities of the same type. Indeed, when a player forms a link, the direct effect of homophily $\beta > 0$ will give incentive to form a link with a person of the same type; the indirect effect is that an increase in the incentive to form links to same type individuals if they have an homogeneous set of friends. If $\beta < 0$ and $\gamma > 0$ is large enough, we could still observe homophily in the aggregate, because the effect of indirect payoffs may offset the direct heterophily. If γ is large enough, it may be profitable to form links to people of different type, as long as their set of friends is homogeneous. This shows that the model has the ability to match many possible outcomes in terms of network configurations.

Negative indirect payoffs. If $\gamma < 0$, there is a form of competition for the indirect friends, and having friends of friends of the same type will decrease utility. In such a setting, we can still observe homophily in aggregate as long as $\beta > 0$ and large enough. If $\beta < 0$, we will have a network displaying aggregate heterophily.

Finally, while the model is able to match complex network configurations in the data, the sign and magnitude of the payoffs is ultimately an empirical question.

3 Estimation Strategy

The estimation recovers the payoffs of the players, that depend on parameters $\theta = (\theta_u, \theta_m, \theta_v)$:

$$u_{ij}(\theta_u) = u(X_i, X_j, \theta_u); \quad m_{ij}(\theta_m) = m(X_i, X_j, \theta_m); \quad v_{ij}(\theta_v) = v(X_i, X_j, \theta_v)$$

Assuming that the observed network is a draw from the stationary distribution of the theoretical model, we can use the distribution in (4) as likelihood of the network data. However, exact maximum likelihood estimation because the distributon $\pi(g, X; \theta)$ is proportional to the normalizing constant $c(\mathcal{G}, X, \theta)$, whose exact evaluation is infeasible, even in small networks. To be concrete, consider a network with $n = 10$ agents. To compute the constant at the current parameter θ we would need to evaluate the potential function for all $2^{90} \backsimeq 10^{27}$ possible networks with 10 agents and compute their sum. Assuming that a state-of-the-art supercomputer can evaluate 10^{12} potential functions in 1 second, it would take almost 40 million years to compute the constant at a single paremeter vector.[11] Therefore direct evaluation of the likelihood is impossible. It is easy to show that the first and second order conditions for the maximum likelihood problem also depend on the normalizing constant. The same problem arises if we use a Bayesian approach and standard Markov Chain Monte Carlo samplers to estimate the posterior

$$p(\theta|g,X) = \frac{\pi(g,X,\theta)\,p(\theta)}{\int_{\Theta}\pi(g,X,\theta)\,p(\theta)\,d\theta}. \tag{7}$$

because equation (7) contains the normalizing constant in the likelihood.

3.1 Estimation Algorithm

To solve this estimation problem, I use the *exchange algorithm*, first developed by Murray et al. (2006) for distribution with intractable normalizing constants and adapted to network models by Caimo and Friel (2011) and Mele (2017*a*). This algorithm uses a double Metropolis-Hastings step to avoid the computation of the normalizing constant $c(\mathcal{G},X,\theta)$ in the likelihood.[12]

While several authors have proposed similar algorithms in the related literature on Exponential Random Graphs Models (ERGM),[13] the models estimated with this methodology typically have very few parameters and use data from very small networks. To the best of my knowledge, this is the first attempt to estimate a high-dimensional model using data from multiple networks. In this section I describe the algorithm for a single network, while in the appendix I provide the extension for multiple independent networks.[14]

The idea of the algorithm is to sample from an augmented distribution using an auxiliary variable. At each iteration, the algorithm proposes a new parameter vector θ', drawn from a suitable proposal distribution $q_\theta(\theta'|\theta)$; in the second step, it samples a network g' (the auxiliary variable) from the likelihood $\pi(g',X,\theta')$; finally, the proposed parameter is accepted with a probability $\alpha_{ex}(\theta,\theta')$, such that the Markov chain of parameters generated by these update rules, has the posterior (7) as unique invariant distribution.

ESTIMATION ALGORITHM
Fix the number of simulations R. At each iteration t, with current parameter $\theta_t = \theta$, network data g and control variables X:

[11] A network with n players has $2^{n(n-1)}$ possible network configurations. The schools used in the empirical section have between 20 and 159 enrolled students. This translates into a minimum of 2^{380} and a maximum of 2^{25122} possible network configurations.

[12] This improvement comes with a cost: the algorithm may produce MCMC chains that have very poor mixing properties (Caimo and Friel, 2011) and high autocorrelation. I partially correct for this problem by carefully calibrating the proposal distribution. In this paper I use a random walk proposal. Alternatively one could update the parameters in blocks or use recent random block techniques as in Chib and Ramamurthy (2009) to improve convergence and mixing.

[13] Caimo and Friel (2011) use the exchange algorithm to estimate ERGM. They improve the mixing of the sampler using the snooker algorithm. Koskinen (2008) proposes the Linked Importance Sampler Auxiliary variable (LISA) algorithm, which uses importance sampling to provide an estimate of the acceptance probability. Another variation of the algorithm is used in Liang (2010).

[14] When the data consist of several independent school networks, I use a parallel version of the algorithm that stores each network in a different processor. Each processor runs the simulations independently and the final results are summarized in the master processor, that updates the parameters for next iteration. Details in Appendix.

1. Propose a new parameter $\theta' \sim q_\theta(\cdot|\theta)$.

2. Simulate R networks from the stationary distribution of the model and collect the last simulated network $g' \sim \mathcal{P}^{(R)}_{\theta'}(g'|g)$, using the following steps (2.1) and (2.2) at each iteration:

   (2.1) At iteration r, with current network g_r and proposed parameter θ', start the simulations at network g and propose a network g^* from a proposal distribution $g^* \sim q_g\left(g^*|g_r\right)$

   (2.2) Update the network: $g_{r+1} = g^*$ with probability $\alpha_{mh}(g_r, g^*)$ and $g_{r+1} = g_r$ with probability $1 - \alpha_{mh}(g_r, g^*)$, where

$$\alpha_{mh}(g_r, g^*) = \min\left\{1, \exp\left[Q(g^*, X, \theta) - Q(g_r, X, \theta)\right] q_g\left(g_r|g^*\right)/q_g\left(g^*|g_r\right)\right\} \tag{8}$$

3. Update the parameter vector: $\theta_{t+1} = \theta'$ with probability $\alpha_{ex}\left(\theta, \theta', g', g\right)$ and $\theta_{t+1} = \theta$ with probability $1 - \alpha_{ex}\left(\theta, \theta', g', g\right)$, where

$$\alpha_{ex}(\theta, \theta', g', g) = \min\left\{1, \frac{\exp\left[Q(g', X, \theta)\right]}{\exp\left[Q(g, X, \theta)\right]}\frac{p\left(\theta'\right)}{p\left(\theta\right)}\frac{q_\theta\left(\theta|\theta'\right)}{q_\theta\left(\theta'|\theta\right)}\frac{\exp\left[Q(g, X, \theta')\right]}{\exp\left[Q(g', X, \theta')\right]}\right\}. \tag{9}$$

The appeal of this algorithm is that all quantities in the acceptance ratio (9) can be evaluated: there are no integrals or normalizing constants to compute. The sampler is likely to accept proposals that move towards high density regions of the posterior distribution, and it is likely to reject proposals that move towards low density regions of the posterior. Therefore, it produces samples of parameters that closely resemble draws from the posterior distribution (7). A formal statement about convergence to the posterior distribution (7) is in Theorem 6 of Mele (2017*a*).[15] In the empirical application we have multiple school networks and therefore the step 2 is parallelized, each school being simulated in a different processor. This speeds up computations and allows inference in complex models.

3.2 Likelihood of multiple independent networks

In the empirical analysis, I specify utility functions linear in parameters. Let $\theta_u = (\theta_{u1}, \theta_{u2}, ..., \theta_{uP})'$, $\theta_m = (\theta_{m1}, \theta_{m2}, ..., \theta_{mL})'$ and $\theta_v = (\theta_{v1}, \theta_{v2}, ..., \theta_{vS})'$ be the utility parameters for u, m and v respectively. Let functions $h : \mathbb{R}^A \times \mathbb{R}^A \to \mathbb{R}$ be defined as

$$u_{ij}\left(\theta_u\right) = \sum_{p=1}^{P} \theta_{up} h_{up}\left(X_i, X_j\right); \quad m_{ij}\left(\theta_m\right) = \sum_{l=1}^{L} \theta_{ml} h_{ml}\left(X_i, X_j\right); \quad v_{ij}\left(\theta_v\right) = \sum_{s=1}^{S} \theta_{vs} h_{vs}\left(X_i, X_j\right)$$

The functions h are quite general. In particular, they can include interactions of multiple control variables, like gender and race; or interactions of individual and network-level attributes, such as race interacted with share of a racial group in the network. In principle, the functions used for direct, mutual and indirect utility may be different. Because of the linearity of u_{ij}, m_{ij} and v_{ij}, the likelihood an exponential family distribution (Lehman (1983))

$$\pi\left(g, X\right) = \frac{\exp\left[\boldsymbol{\theta}'\boldsymbol{t}\left(g, X\right)\right]}{\sum_{\omega \in \mathcal{G}} \exp\left[\boldsymbol{\theta}'\boldsymbol{t}\left(\omega, X\right)\right]}, \tag{10}$$

[15] Practical implementation of the algorithm is discussed in Mele (2017*a*).

where $\boldsymbol{\theta} = (\theta_u, \theta_m, \theta_v)'$ is a vector of parameters and $\boldsymbol{t}(g, X) = (t_1(g, X), ..., t_K(g, X))$ is a vector of *sufficient statistics* for the network formation model, such as number of links, the number of links among black students, number of reciprocated links, etc.[16]

The estimation is performed with a sample of school network data. For each school we observe the friedship nominations g_c and student's control variables X_c, for $c = 1, ..., C$. The nature of the sample and the the survey we use (see section about Add Health data) makes the school networks independent. Therefore the log-likelihood of our data is the sum of the log-likelihoods of each school

$$\ell(g, X) = \sum_{c=1}^{C} \boldsymbol{\theta}'\boldsymbol{t}(g_c, X_c) - \kappa(\mathcal{G}, X, \boldsymbol{\theta}) \tag{11}$$

where the log-normalizing constant $\kappa(\mathcal{G}, \mathcal{X}, \theta)$ is

$$\kappa(\mathcal{G}, X, \boldsymbol{\theta}) = \sum_{c=1}^{C} \log \left(\sum_{\omega_c \in \mathcal{G}_c} \exp\left[\boldsymbol{\theta}'\boldsymbol{t}(\omega_c, X_c)\right] \right) \tag{12}$$

This is the log-likelihood of independent observations from an exponential family (Lehman (1983), Wainwright and Jordan (2008), Snijders (2002), Geyer and Thompson (1992)).

3.3 Identification

The model generates a dynamic sequence of networks, converging to stationarity. In principle, if the researcher has access to data where the same network is observed over time, we can estimate and identify the transition probability of the Markov Chain, that is the meeting probability and the conditional linking probability implied by the payoffs and extreme value distributed and independent shocks.

However, the data consists of only one observation at a single point in time. With such data, we cannot identify the transition probability of the model. The crucial assumptions to get identification of the stationary distribution are that shocks are independent extreme value and the meeting process does not depend on the existence of a link between two players. These assumptions make the stationary distribution independent of the specific meeting process, while allowing us to write it in closed-form as an exponential family distribution.

This allows me to identify the preferences of individuals from a network drawn from the stationary distribution. My assumptions make sure that the conditional link probability of each player is consistent with the likelihood of the whole network (Tamer, 2003). This is achieved by the assumptions on the payoff functions that guarantee the existence of a potential.

Since the likelihood of the model belongs to the exponential family (Lehman, 1983), when we observe a sample of independent networks (at a single point in time) the parameters are identified

[16] This likelihood defines an exponential random graph model (ERGM), a statistical model of networks used in many applications (Snijders (2002),Frank and Strauss (1986), Moody (2001), Boucher and Mourifie (forthcoming),DePaula (forthcoming), Chandrasekhar (2016)). My theoretical model can be interpreted as providing the microfoundations for exponential random graphs. In this sense, we can interpret the ERGM as the stationary equilibrium of a strategic game of network formation, where myopic agents follow a stochastic best response dynamics and utilities are linear functions of the parameters.

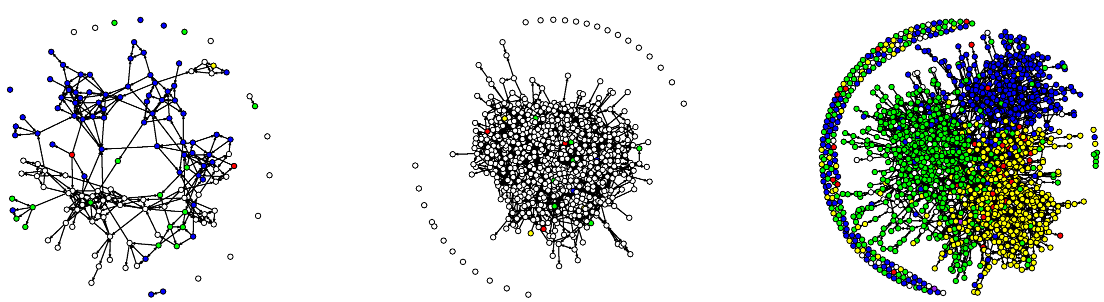
through the variation of the sufficient statistics across networks. If the sufficient statistics are not linearly dependent, then the exponential family is minimal and the likelihood is strictly concave, therefore the mode is unique (Geyer and Thompson, 1992; Lehman, 1983; Wainwright and Jordan, 2008). Our priors are relatively flat, so most of the information about the posterior is given by the likelihood.

As an aside, notice that identification may still fail when the observed sufficient statistics are "extreme". As a simple example, consider the case in which the network does not have any reciprocated links or does not have any links between hispanics. In that case it will be impossible to estimate the effect of mutual links and the homophily of hispanic students.

While in principle it is possible to add unobserved heterogeneity in the model, it is not clear that we can identify such effects using only one network observation (DePaula, forthcoming; Graham, 2017; Chandrasekhar, 2016). The literature on latent position network models suggests that in some special case this is possible (Airoldi et al., 2008), however such models consider links that are conditionally independent, while in our model the links are conditionally dependent.

Figure 1: Three school friendship networks from Add Health, saturated sample

white=Whites; blue = African Americans; yellow = Asians; green = Hispanics; red = Others



A. School 28 B. School 58 C. School 77

Note: The graphs represent the friendship network of a school extracted from AddHealth. Each dot represents a student, each arrow is a friend nomination. The colors represent racial groups.

3.4 The Add Health Data

The *National Longitudinal Study of Adolescent Health* (Add Health) is a dataset containing information on a nationally representative sample of US schools. The survey started in 1994, when the 90118 participants were entering grades 7-12, and the project collected data in four successive waves.[17] Each student responded to an *in-school* questionnaire, and a subsample of 20745 was given an *in-home* interview to collect more detailed information about behaviors, characteristics and health status. In this paper I use only data from the *saturated sample* of Wave I, containing information on 16 schools. Each student in this sample completed both the in-school and in-home

[17] More details about the sampling design and the representativeness are contained in Moody (2001) and the Add Health website http://www.cpc.unc.edu/projects/addhealth/projects/addhealth

questionnaires, and the researchers made a significant effort to avoid any missing information on the students.[18]

Most schools in the saturated sample are relatively small and enroll about 100 students, and the largest schools enroll 1664, 811 and 159 students. The final sample includes 3604 students in 16 different schools. I also provide estimates excluding the two largest schools, for which the sample is 1129 students.

The *in-school* questionnaire collects the social network of each participant. A sub-sample of 20745 students was also given an *in-home* questionnaire, that collected most of the sensible data, in addition to the network data. Each student was given a school roster and was asked to identify up to five male and five female friends.[19] I use the friendship nominations as proxy for the social network in a school. The resulting network is *directed*: Paul may nominate Jim, but this does not necessarily imply that Jim nominates Paul.[20] The model developed in this paper takes this feature of the data into account. In Figure 1 we report three school networks from the saturated sample, where each dot is a student, its color represent the racial group and an arrow is a friendship nomination.

In the empirical specification I control for racial group, grade and gender. A student with a missing value in any of these variables is dropped from the sample. Each student that declares to be of Hispanic origin is considered Hispanic. The remaining non-Hispanic students are assigned to the racial group they declared. Therefore the racial categories are: White, Black, Asian, Hispanic and Other race. Other race contains Native Americans. I include controls for parental income, using a question from the parent questionnaire.[21]

There may be some unobservable variables that affect network formation. For example some students may be "cool" and receive more friendship links than others. Unobserved heterogeneity can be included in the model at the cost of significant additional computational burden in forms of random effects for each node.[22] To partially control for such unobserved determinants of link formation, I use information from the interviewer remarks about the *physical attractiveness* and *personality* of the student interviewed. I define a dummy variable "beauty", which is equal to 1 if the interviewer reported that the students was very attractive. Analogously, the dummy "personality" is equal to 1 if the interviewer reported that the personality of the student was very attractive. Finally, I include school dummies to control for different schools characteristics.

Descriptive statistics are in Table 1. There is a certain amount of variation in the number of links: some schools are more social and form many links per capita, while other schools have

[18]While this sample contains no missing covariate information for the students, there are several missing values for the parental variables.

[19]In the in-home interviews there are 20745 students. Of these, about 5% nominated 5 male friends and 5% nominated 5 female friends. Only about 3% of the 20745 students nominated both 5 males and 5 females friends (Moody, 2001).

[20]Some authors do not consider this feature of the data and they recode the friendships as mutual: if a student nominates another one, the opposite nomination is also assumed.

[21]There are several cases in which the family income is missing. For those observations, I imputed values drawn from the unconditional income distribution of the community. An alternative but computationally very costly alternative is to introduce an additional step in the simulation, in which the imputation of missing incomes is done at each iteration. Given the computational burden of my estimation exercise I did not pursue this alternative here.

[22]See Mele (2017*a*) for a discussion. Badev (2013) also discusses the introduction of random effects. Mele (2017*b*) includes unobserved heterogeneity in the form of unobserved communities. The computational burden of the exchange algorithm for such model is impractical for estimation of networks with more than 200 nodes.

Table 1: **Descriptive Statistics for the schools in the Saturated Sample**

School	1	2	3	7	8	28	58	77	81	88	106	115	126	175	194	369
Students	44	60	117	159	110	150	811	1664	98	90	81	20	53	52	43	52
Links	12	120	125	344	239	355	3290	3604	163	308	162	44	123	171	42	48
Females	0.5	0.517	0.419	0.44	0.5	0.587	0.473	0.483	0.531	0.522	0.531	0.55	0.491	0.538	0.512	0.654
Clustering	0.000	0.421	0.154	0.222	0.282	0.291	0.197	0.193	0.244	0.362	0.202	0.393	0.392	0.284	0.064	0.056
Density	0.006	0.034	0.009	0.014	0.020	0.016	0.005	0.001	0.017	0.038	0.024	0.116	0.045	0.064	0.023	0.018
								A. Racial Composition								
Whites	0.5	0.95	0.983	0.981	0.973	0.42	0.978	0.055	0.98	0.989	0	1	0.472	0.769	0.977	0.942
Blacks	0.136	0	0	0.006	0.018	0.453	0.002	0.233	0	0	0.963	0	0.151	0.019	0	0
Asians	0	0	0	0	0.009	0.007	0.005	0.299	0.01	0	0	0	0.038	0.038	0	0
Hispanics	0.364	0.05	0.017	0.006	0	0.107	0.011	0.392	0.01	0	0.025	0	0.302	0.154	0.023	0.058
Others	0	0	0	0	0	0.013	0.004	0.02	0	0.011	0	0	0.038	0.019	0	0
Racial Fragm	0.599	0.095	0.034	0.037	0.053	0.606	0.044	0.699	0.04	0.022	0.072	0	0.661	0.382	0.045	0.109
								B. Grade Composition								
7th Grade	0.159	0.2	0.128	0.145	0.227	0.173	0.002	0.001	0.112	0.144	0.506	0.4	0.491	0.462	0.488	0.538
8th Grade	0.159	0.217	0.154	0.157	0.2	0.173	0.004	0.003	0.153	0.178	0.481	0.6	0.472	0.538	0.488	0.462
9th Grade	0.114	0.2	0.12	0.214	0.136	0.2	0.289	0.004	0.153	0.122	0.012	0	0.038	0	0	0
10th Grade	0.273	0.133	0.205	0.157	0.182	0.167	0.277	0.346	0.214	0.167	0	0	0	0	0	0
11th Grade	0.136	0.167	0.179	0.164	0.118	0.14	0.223	0.345	0.265	0.211	0	0	0	0	0.023	0
12th Grade	0.159	0.083	0.214	0.164	0.136	0.147	0.205	0.301	0.102	0.178	0	0	0	0	0	0
								C. Segregation								
FSI gender	0.348	0.035	0.095	0.263	0.100	0.206	0.142	0.228	0.196	0.107	0.186	0.123	0.095	0.050	0.186	0.000
FSI race	0.000	0.689	0.180	0.553	0.000	0.671	0.014	0.690	0.819	0.816	0.000		0.403	0.000	0.000	0.560
FSI income 90	0.596	0.332	0.000	0.189	0.000	0.118	0.024	0.000	0.000	0.077	0.000	0.000	0.000	0.272	0.000	0.384
FSI income 50	0.023	0.000	0.027	0.133	0.000	0.013	0.077	0.082	0.064	0.116	0.012	0.000	0.269	0.069	0.131	0.000
SSI gender	0.305	0.541	0.493	0.697	0.586	0.659	0.798	0.727	0.614	0.618	0.601	0.658	0.561	0.696	0.488	0.461
SSI race	0.146	0.862	0.791	0.894	0.865	0.754	0.927	0.748	0.805	0.921	0.761		0.550	0.632	0.786	0.817
SSI income 90	0.409	0.767	0.641	0.783	0.735	0.803	0.820	0.767	0.684	0.836	0.705	0.778	0.726	0.825	0.709	0.681
SSI income 50	0.214	0.469	0.444	0.601	0.501	0.547	0.726	0.620	0.439	0.563	0.460	0.419	0.535	0.611	0.484	0.421

Descriptive statistics of the saturated sample for Wave 1 of Add Health.

very few friendship nominations. The ratio of boys to girls is quite balanced in almost all schools, except school 369, where female students are a larger majority.

Panel A summarizes the racial composition. Many schools are almost racially homogeneous. School 1, 28, 77, 126 and 175 are more diverse as reflected in the Racial Fragmentation index (Alesina and Ferrara (2000), Alesina et al. (1999)), an index that measures the probability that two randomly chosen students in the school belong to different racial groups.[23] An index of 0 indicates that there is only one racial group and the population is perfectly homogeneous. Higher values of the index represents increasing levels of racial heterogeneity. Panel B summarizes the grade composition. Most schools offer all grades from 7th to 12th, with homogeneous population across grades. Several schools only have lower grades.

Panel C analyzes the racial, gender and income segregation of each school. Income segregation is measured in two ways: the segregation of the students in the 90th percentile from the rest and the segregation of students above median income from the rest. The measures of segregation are the Freeman Segregation Index (FSI) (Freeman, 1972) and the Spectral Segregation Index (SSI) (Echenique and Fryer, 2007). The FSI varies from 0 to 1 and measures the difference between the expected and actual number of links among individuals of different groups. An index of 0 means that the actual network closely resembles one in which links are formed at random. Higher values indicate more segregation. The index varies between 0 and 1, where the maximum corresponds to a network in which there are no cross-group links. The SSI computes segregation usin spectral methods. The intuition is as follows: first order segregation of a student is the share of a student's social interations with students in their own group; second order segregation is the average over the student's own group of first order segregation; n-th order segregation is the average of the students's own group of n-1-th segregation. The student's SSI is the limit for n going to infinity of this sequence. We take the average students SSI in a school as measure of segregation in the school. SSI also varies from 0 to 1 as segregation increases.

The schools are quite segregated based on race, as long as there is some diversity in the school. Income segregation and gender segregation seem to be lower in these schools. We notice that the SSI measures high segregation in schools that are almost homogeneous by race, because the largest connected component of the network contains only individuals of the same group. However, FSI measures very low segregation when the school is homogeneous. We use both measures to understand the effects of our counterfactual simulations in the next section.

4 Empirical Results

4.1 Posterior estimates

In Table 2 we report the estimated posterior means for 6 models. More detailed tables including posterior mean, median, standard deviation and 95% credible intervals are in Appendix. All the estimated posterior means have small credible intervals, therefore their sign and magnitude is estimated very precisely.[24]

The models in columns (1)-(4) are estimated with the smaller 14 schools in the saturated sample. This sample includes a total of 1129 students and 112,751 pairs of potential links. The latter

[23] If there are K racial groups and the share of each race is s_k, the index is $FRAG = 1 - \sum_{k=1}^{K}(s_k)^2$.

[24] All the replication codes are availabile at https://github.com/meleangelo/segnet.

should be thought of as the number of observations of our model, since we are modeling the links probabilities and decisions. Columns (5) and (6) are estimated with the full saturated sample including 16 schools. This includes the two largest schools with 811 and 1664 students, increasing our student sample to 3604 and the number of dyads to 3,536,893. In models (1), (3) and (5) preferences are modeled as function of the directed utility only, therefore omitting the externalities payoffs; in models (2), (4) and (6) we include all the payoffs and estimate the preferences for reciprocity and indirect links. All the models control for other school characteristics with a set of school dummies.

Each estimate measures the marginal effect of the variable on the payoff: for example, the parameter associated with the direct utility of WHITE-WHITE measures the marginal utility of a white student when forming a link to another white student, other things being equal.

Model (1) does not include any externality, thus being a standard (Bayesian) logistic regression estimate. Model (2) has the same specification as (1), but includes mutual and indirect utility in the specification. The signs of the coefficients for the direct utility seem to agree among the two specifications, except in three cases: once we control for mutual and indirect utility, homophily by gender (SAME GENDER), physical attractiveness of the alter (ATTRACTIVE j (Phys)) and the effect of white share in the school (SHARE WHITES) change sign. We notice that the signs in model (2) are consistent with the remaining specifications (3)-(6).

The coefficient for racial homophily are positive in both specifications (WHITE-WHITE, BLACK-BLACK, HISP-HISP). In model (2) the payoff from mutual links display racial homophily as well, while for indirect links there is mixed evidence, as there is a negative coefficient for BLACK-BLACK payoff.

Models (3)-(6) are estimated according to our favorite specification. In these models we control for some demographics of the student nominating friends: gender, race and income of the parents (in logs).

A comparison of models (3) and (5), as well as (4) and (6) shows that the sample with the 14 small schools and the larger sample including all the 16 schools of the saturated sample generate qualitatively similar estimates, while the magnitudes of the parameters are different. Because model (6) is estimated with more than 3.5 million observations (the number of possible student pairs) we are more confident about these estimates and discuss these in detail.

In model (6), the estimated preferences show racial homophily for direct and indirect links, except for hispanic students. For mutual links, only blacks seem to show homophily. Hispanics have the highest propensity to form new links, but interestingly prefer links outside their own racial group, especially when the share of hispanic students in the school is relatively large. The mutual links constant is positive, showing that reciprocity provides positive baseline utility, but the constant for the indirect friends payoff is negative, suggesting congestion or competition for indirect friends. High income reduces the willingness to form links, while males seems to be more active than females in friendship formation.
An increase in the share of blacks and hispanic students will increase willingness to link, while the opposite happens with the share of caucasian students. White and black students homophily is higher in schools with a higher share of whites and blacks. This is important as different groups will possibly have different responses to desegregation efforts: some group may engage more in interracial friendships, while some group may segregate even more.

Physically attractive students do not initiate friendships often, but receive friendship nomina-

Table 2: Posterior mean of estimated models

	(1)	(2)	(3)	(4)	(5)	(6)
			A. Direct utility (u_{ij})			
CONSTANT	-6.9201	-5.5381	-6.6500	-5.9132	-7.2182	-5.8070
MALE i			-0.1517	0.0463	-0.2718	0.2350
WHITE i			-0.1710	0.0044^a	0.0440^a	0.3023
BLACK i			1.0451	1.1310	0.7074	1.1801
HISP i			2.0990	2.2806	1.4590	2.0295
INCOME i (logs)			-2.0543	-1.6492	-1.8738	-1.4645
SAME GENDER	-0.4545	0.1850	0.2067	0.4851	0.3154	0.7644
SAME GRADE	2.3124	2.2384	2.3817	2.0113	2.5185	2.1800
WHITE-WHITE	0.3504	0.5414	1.0138	0.5720	0.9959	0.2739
BLACK-BLACK	0.1443	0.3660	1.6491	1.1445	1.5347	0.9405
HISP-HISP	1.8597	1.6794	0.3186	-0.2269	0.7130	-0.1394
ATTRACTIVE i (Phys)	0.2757	0.3068	-2.3568	-2.2413	-1.9291	-1.9430
ATTRACTIVE j (Phys)	-0.0410	0.2322	2.5166	1.5861	2.7615	1.2609
ATTRACTIVE i (Pers)	-0.4402	0.0063^a	-0.4964	-0.1570	-0.8646	-0.1631
ATTRACTIVE j (Pers)	1.0672	0.8678	-1.0932	-0.7390	-0.6361	-0.3939
INCOME i - INCOME j (logs)	0.1793	0.1462	0.8883	0.9012	0.9938	0.7403
INCOME i + INCOME j (logs)	-0.0882	-0.0806	1.0947	0.9244	0.8977	0.6892
SHARE WHITES	0.9070	-0.4814	-1.7088	-1.4420	-1.5748	-1.6126
SHARE BLACKS	3.2238	3.0985	1.3416	1.8309	0.7645	1.9618
SHARE HISP	2.524	2.444	0.8397	0.7798	1.0078	0.7731
WHITE-WHITE * SHARE WHITES	1.3962	1.0094	4.3915	2.7840	4.7269	2.3272
BLACK-BLACK * SHARE BLACKS	0.4664	0.1478	0.2528	0.4028	0.1172	0.2516
HISP-HISP * SHARE HISP	-1.5643	-1.4255	-1.6908	-1.3630	-1.3872	-1.1400
			B. Mutual utility (m_{ij})			
CONSTANT		1.1853		6.1668		5.3139
SAME GENDER		1.1652		1.0716		1.1539
SAME GRADE		-1.6882		-3.0514		-3.0575
WHITE-WHITE		0.0073^a		-0.6017		-0.4960
BLACK-BLACK		0.7468		1.1177		0.7067
HISP-HISP		0.7779		-1.4659		-1.4639
			C. Indirect utility and Popularity (v_{ij})			
CONSTANT		-0.2891		-0.4705		-0.4308
SAME GENDER		0.1721		-0.4074		-0.3987
SAME GRADE		-0.3145		0.1136		0.3266
WHITE-WHITE		0.2239		0.1856		0.2978
BLACK-BLACK		-0.1364		0.1372		0.1202
HISP-HISP		0.4328		-0.5067		-0.2859
SCHOOL DUMMIES	YES	YES	YES	YES	YES	YES
			D. Sample size			
# Schools	14	14	14	14	16	16
# Students	1129	1129	1129	1129	3604	3604
# Pairs/Dyads	112,751	112,751	112,751	112,751	3,536,893	3,536,893

Models (1)-(4): posterior sample of 100,000 parameter and 5000 network simulations per parameter. Models (5)-(6): posterior sample of 20,000 parameter and 10,000 network simulations per parameter. [a] credible 95% interval contains both positive and negative values.

tions with higher frequency. Students with attractive personalities form and receive fewer nominations. Income differences increase the likelihood of friendship, suggesting that students tend to mix based on income levels; higher income levels of the pair involved in the link increase the number of friendships formed. The total number of friends is higher in schools with higher fraction of minorities.

4.2 Policy Experiments

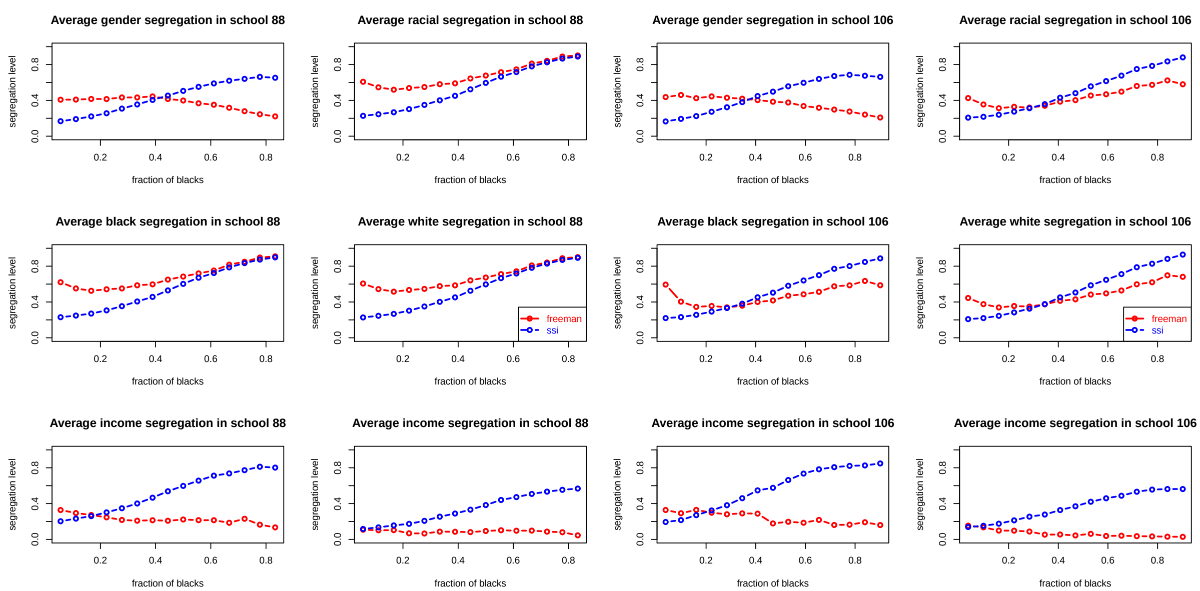
In this section the estimated model is used to estimate the effect of counterfactual policies on several network characteristics. I simulate 1000 equilibrium networks for each policy change, using the posterior mean estimated in Table 2, column (6). For each network, I compute measures of segregation, clustering and centrality. [25]

4.2.1 Reallocation of students based on race

Schools 88 and 106 are very racially homogeneous, being almost completely white and black, respectively. In these simulations, a total of x white students is moved from school 88 to school 106; viceversa, the same number x of black students is moved from school 106 to school 88. This policy changes the racial heterogeneity levels within schools by directly targeting students based on their race. Our model predicts that in equilibrium the students will best respond to the new environmnet by modifying their linking strategy.

In Figure 2 we show the effect of this policy on the average segregation in the schools. We have computed segregation beased on race, gender and income. In all graphs, the red line represents the Freeman Segregation Index (FSI) of Freeman (1972), the blue is the Spectral Segregation Index (SSI) of Echenique and Fryer (2007).

Figure 2: Average segregation in school 88 and 106



Notes: each policy simulation simulates the network using 1000 draws from the posterior distribution estimated in Model (6) of Table 2.

Let's first consider the FSI (in red in Figure 2). In school 88, as the policy increases diversity by increasing the fraction of blacks, racial segregation first decreases and then starts increasing again. The intuition for this outcome is that when the fraction of minority students in the school is small enough, they need to find friends in another racial group. However, when the size of the

[25] The complete simulation codes are available in Github.

minority in the school is large enough the homophily effect prevails and minorities will tend to self-segregate.

The results on gender segregation suggest that the segregation would increase slightly as the fraction of black students increases and then decrease.

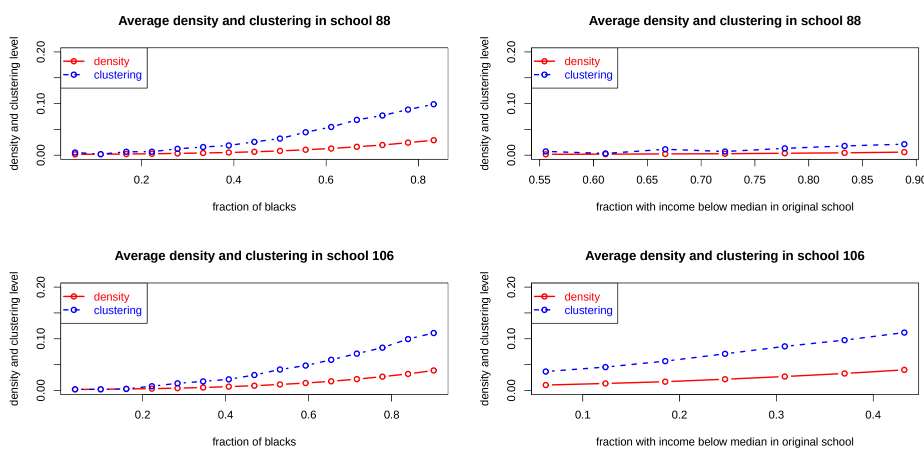
These results based on the Freeman Segregation Index, suggest that a policy promoting perfect racial integration among schools, would not necessarily increase interactions among students of different racial background.

On the other hand, the results using the Spectral Segregation Index (SSI, in blue in Figure 2, seem to point to a slightly different conclusion.

The result for the SSI should be thought as what happens to the average student in the school. This index says that at least in term of the SSI, the average student will be exposed to higher gender, race and income segregation as the fraction of blacks in the school increases. According to this measure of segregation, a perfectly integrated school would be a good compromise.

This policy has an effect on several structural characteristics of the network. Figure 3 (left panels) shows the density and transitivity of the network as the diversity of the schools changes. An increase in the fraction of blacks will increase both density and clustering in both schools.

Figure 3: Density and Clustering



Notes: each policy simulation simulates the network using 1000 draws from the posterior distribution estimated in Model (6) of Table 2.

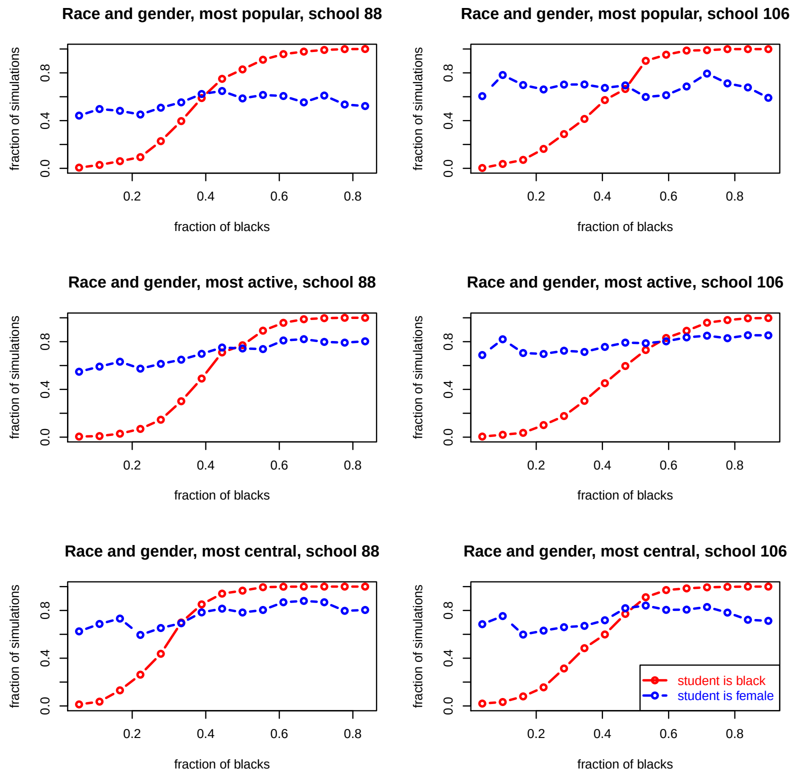
Figure 4 shows race and gender of the most central students as measured by three indicators: 1) indegree centrality measures popularity; 2) outdegree centrality measures social activity; 3) eigenvector centrality measures the most crucial node. These indicators are usually correlated with performance(Calvo-Armengol et al., 2009). The graphs on the top compare schools based on indegree centrality, in the middle we have outdegree centrality and at the bottom the eigenvector centrality.

In many cases the most popular person is a girl, in both schools.

The probability that the most central students is black increases with the fraction of blacks in the school. This pattern seems to be similar in both schools and across the centrality indicators.

According to our most intuitive measure of segregation, the Freeman Segregation Index (FSI), the conclusion of this experiment is that trying to make schools equally diverse by equalizing their racial share may not decrease segregation necessarily. Indeed it seems better to have a relatively

Figure 4: Gender and Race of the most central



Notes: each policy simulation simulates the network using 1000 draws from the posterior distribution estimated in Model (6) of Table 2.

small minority, that will not be able to self-segregate. While the patterns of segregation as we increase the fraction of blacks in the school look the same, there are differences in the magnitude of segregation across schools, due to the difference in which whites and blacks respond to changes in the fraction of their own groups in the school. Income segregation slightly decreases with the reallocation of blacks to school 88, while it increases for school 106. According to the Spectral Segregation Index (SSI), an increase in the fraction of blacks will always increase segregation. This means that a policy that makes schools equally diverse will be a good compromise in terms of racial and income segregation.

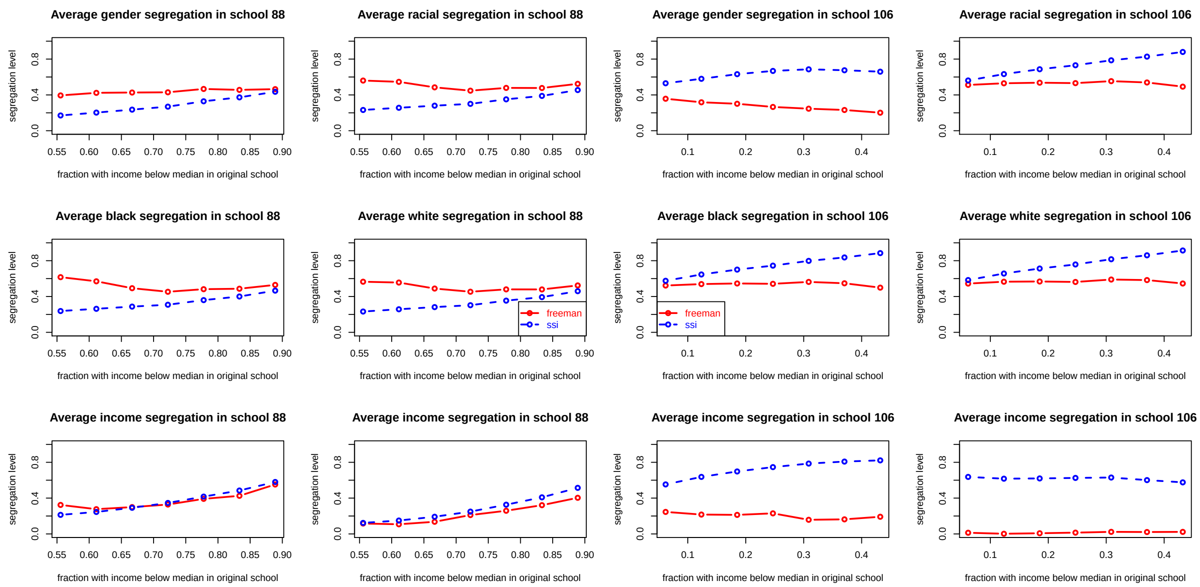
The policy will increase density and clustering in school 88, while decreasing both in school 106. If schools are equally racially diverse, black students will be on average more popular and central. This is also true if whites are a small minority.

4.2.2 Reallocation of students based on income

The second policy experiment consists of swapping students based on their parental income. For both school 88 and 106, we compute the median income of the parents, and we swap students of school 88 with incomes above median with students of school 106 with incomes below median.

This procedure increases segregation by income across the schools, making them more homogeneous by income. School 88 becomes populated by poorer students, while school 106 becomes richer. On the other hand, this policy increases racial diversity within schools, because we are swapping white/caucasian students from school 88 with black students from school 106, thus making schools more heterogeneous by race.

Figure 5: Average segregation in school 88 and 106



Notes: each policy simulation simulates the network using 1000 draws from the posterior distribution estimated in Model (6) of Table 2.

In Figure 5 we show the effect of this policy on the average segregation in school 88 and 106. To interpret the results, notice that for school 88, moving left to right on the x-axis makes the school *more homogeneous by income (and poorer) and more diverse by race*. Viceversa, for school 106, moving left to right on the x-axis makes the school *more heterogeneous by income and less diverse by race*.

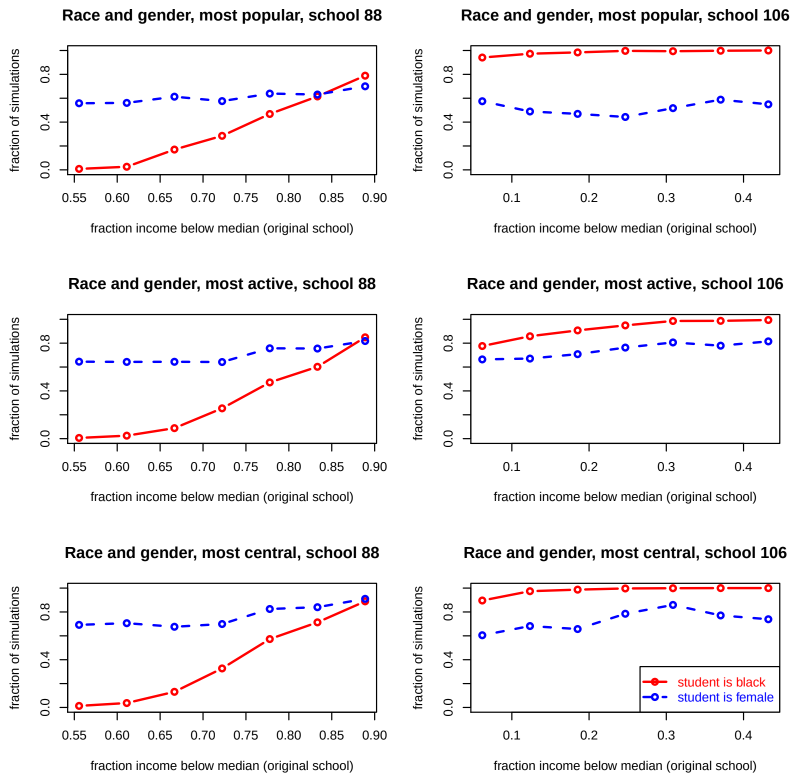
According to the FSI, for school 88 there is a moderate effect on racial segregation, that follows the similar U-shaped pattern we have seen in the previous race-based policy. Income segregation increases with income homogeneity and racial heterogeneity. This reflects the fact that preferences estimated in model 6 of Table 2 show heterophily in income levels. On the other hand, in school 106 there is no effect in income segregation, and gender segregation decreases with income homogeneity and racial diversity.

The SSI index of segregation mirrors the patterns of the policy with re-allocation based on race.

In Figure 3 (right panels) we show the density and transitivity of the network. While school 88 does not show any substantial change, school 106's density and clustering increase with income homogeneity and racial diversity. In this school there is no change in which racial group is most central as a consequence of the policy (see Figure 6, right panel). On the other hand, for school 88 the increase in income homogeneity also increases the popularity of black students.

In summary, this policy increases segregation by income across the schools, making income distribution more homogeneous within schools and simultaneusly increasing racial diversity. This has asymmetric effects in the two schools racial segregation, as measured by the FSI.

Figure 6: Gender and Race of the most central



Notes: each policy simulation simulates the network using 1000 draws from the posterior distribution estimated in Model (6) of Table 2.

5 Conclusions

This paper analyzed racial segregation in schools, guided by an equilibrium model of network formation. The model generates segregation as an equilibrium outcome and allows me to estimate the preferences of students that include direct effects of observables and indirect payoffs for reciprocated relationships and indirect friends. I find homophily by race, both in direct and indirect links. My specification allows homophily to vary with the fraction of the racial groups in the school: an increase in the fraction of white students decreases the propensity of white students to form links within the same racial group; the opposite holds for hispanics or blacks. These differences are important to understand the effect of policies that modify the relative shares of groups in the school. There is evidence of homophily beyond direct links, thus supporting a model that includes indirect effects of linking in the preferences.

To illustrate the implications of such estimated preferences, I simulate two “desegregation” policies using two racially homogeneous schools from my sample, where students populations are almost completely white and black, respectively. The model simulations provide predictions about the expected levels of segregation and network features implied by swapping students among schools.

The first policy simulation swaps black students from one school with white students from the other school, thus modifying the racial balance within school. Interestingly the outcomes of these policy vary by the way we measure segregation. According to the Freeman Segregation Index, integrating the two schools may potentially increase segregation. However, according to the Spectral Segregation Index, perfect integration across the schools leads to the best compromise in terms of racial segregation.

The second simulation decreases income heterogeneity within schools by swapping kids with parental income above median in a school with students whose parental income is below median in the other school. This simultaneously increase racial diversity within schools. According to the FSI there is no much effect on the racial segregation in both schools. However, income segregation increases in the school that becomes poorer. In the latter school the popularity of black students increases with income homogeneity (and racial diversity).

These results suggest that desegregation policies do not necessarily lead to more interactions among students of different ethnic or socioeconomic background.

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A Computational Details

The linear specification allows for utility functions involving network-level controls, when estimation is performed using multiple networks. This can be achieved by a specification of the parameters such as

$$\theta_p = \theta_{p0} + \sum_{c=1}^{C} \theta_{pc} Z_c \tag{13}$$

where Z_c is a network-level variable. The estimation methodology presented above can be applied to this specification without any change. However, estimation of a model with unobserved heterogeneity would require significant additional computational effort (see Appendix C in Mele (2017*a*)).

I choose somewhat vague priors for the parameters to extract most of the information from the data. I assume independent normal priors

$$p(\theta) = \mathcal{N}(\mathbf{0}, 3\mathbf{I}_P), \tag{14}$$

where P is the number of parameters.

The proposal distribution for the posterior simulation is

$$q_\theta(\cdot|\theta) = \mathcal{N}(\mathbf{0}, \delta\mathbf{\Sigma}), \tag{15}$$

where δ is a scaling factor and $\mathbf{\Sigma}$ is a covariance matrix.

I use an adaptive procedure to determine a suitable $\mathbf{\Sigma}$. I start the iterations with $\mathbf{\Sigma} = \lambda\mathbf{I}_P$, where λ is a vector of standard deviations. I choose λ so that the sampler accepts at least 20%-25% of the proposed parameters, as is standard in the literature (Gelman et al., 2003; Robert and Casella, 2005). I run the chain and monitor convergence using standard methods. Once the chains have reached approximate convergence, I estimate the covariance matrix of the chains and use it as an approximate $\mathbf{\Sigma}$ for the next set of simulations. The scaling factor is $\delta = 2.38^2/P$ as suggested in Gelman et al. (1996).

The network sampler uses a proposal $q_g(g|g')$, that selects a link to be updated at each period according to a discrete uniform distribution. The probability of network inversion is $p_{inv} = 0.01$.

All the posterior distributions shown in the following graphs are obtained with a simulation of

100000 Metropolis-Hastings updates of the parameters. These simulations start from values found after extensive experimentation with different starting values and burn-in periods, monitoring convergence using standard methods. For each parameter update, I simulate the network for 5000 iterations to collect a sample from the stationary distribution. For the estimation with 16 schools I ran 20000 parameter simulations and 10000 network simulations for each parameter.

A.1 Parallel estimation with multiple networks

When data from multiple independent networks are available the estimation routines are easily adapted. Assume the researcher has data from C networks: let g_c and X_c denote the network matrix and the individual controls for network c, $c = 1, ..., C$. The aggregate data are denoted as $g = \{g_1, ..., g_c\}$ and $X = \{X_1, ..., X_c\}$.

Assuming each network is drawn from the stationary equilibrium of the model, each network has distribution

$$\pi(g_c, X_c, \theta) = \frac{\exp[Q(g_c, X_c, \theta)]}{\sum\limits_{\omega \in \mathcal{G}_c} \exp[Q(\omega_c, X_c, \theta)]} \tag{16}$$

Since each network is independent, the likelihood of the data (g, X) can be written as

$$\begin{aligned} \pi(g, X, \theta) &= \prod_{c=1}^{C} \pi(g_c, X_c, \theta) = \prod_{c=1}^{C} \left\{ \frac{\exp[Q(g_c, X_c, \theta)]}{c(\mathcal{G}_c, X_c, \theta)} \right\} \\ &= \frac{\exp\left[\sum_{c=1}^{C} Q(g_c, X_c, \theta)\right]}{\prod_{c=1}^{C} c(\mathcal{G}_c, X_c, \theta)} = \frac{\exp\left[\sum_{c=1}^{C} Q(g_c, X_c, \theta)\right]}{\mathcal{C}(\mathcal{G}, X, \theta)} \end{aligned}$$

where $\mathcal{G} = \bigcup_{c=1}^{C} \mathcal{G}_c$ and $X = \{X_1, ..., X_C\}$. The likelihood for multiple independent networks is of the same form as the likelihood for one network observation. The structure of this likelihood makes parallelization extremely easy: each network can be simulated independently using the network simulation algorithm; at the end of the simulation we collect the last network and compute the potential; then we compute the sum of potentials and use it to compute the probability of update. Therefore, the algorithm is modified as follows

ALGORITHM 1. ***(Parallel approximate exchange algorithm***
Fix the number of simulations R. Store each network data (g_c, X_c) in a different processor/core. At each iteration t, with current parameter $\theta_t = \theta$ and network data g

1. *Propose a new parameter θ' from a distribution $q_\theta(\cdot|\theta)$*

$$\theta' \sim q_\theta(\cdot|\theta) \tag{17}$$

2. *For each processor c, start the network sampler at the observed network g_c, iterating for R steps using parameter θ' and collect the last simulated network g'_c*

$$g'_c \sim \mathcal{P}_{\theta'}^{(R)}(g'_c|g_c) \tag{18}$$

3. Update the parameter according to

$$\theta_{t+1} = \begin{cases} \theta' & \textit{with prob. } \alpha_{pex}(\theta, \theta') \\ \theta & \textit{with prob. } 1 - \alpha_{pex}(\theta, \theta') \end{cases}$$

where

$$\alpha_{pex}(\theta,\theta') = \min\left\{1, \frac{\exp\left[\sum_{c=1}^{C} Q(g'_c, X_c, \theta)\right]}{\exp\left[\sum_{c=1}^{C} Q(g_c, X_c, \theta)\right]} \frac{p(\theta')}{p(\theta)} \frac{q_\theta(\theta|\theta')}{q_\theta(\theta'|\theta)} \frac{\exp\left[\sum_{c=1}^{C} Q(g_c, X_c, \theta')\right]}{\exp\left[\sum_{c=1}^{C} Q(g'_c, X_c, \theta')\right]}\right\} \quad (19)$$

The speed of the algorithm depends on the largest network in the data. Since each parameter update requires the result of each processor simulation there is some idle time, since small networks are simulated much faster. However, one could easily modify the algorithm to have different number of network simulations for networks of different sizes, so for each c we would have a different R_c

B Freeman Segregation Index

The Freeman segregation index measures the degree of segregation in a population with two groups (Freeman, 1972). Assume there are two groups, A and B. Let n_{AB} be the total number of links that individuals of group A form to individuals of group B. Let n_{BA}, n_{BB} and n_{AA} be analogously defined. The original index developed by Freeman (1972) is defined as

$$FSI = \frac{\mathbb{E}[n_{AB}] + \mathbb{E}[n_{BA}] - (n_{AB} + n_{BA})}{\mathbb{E}[n_{AB}] + \mathbb{E}[n_{BA}]} \quad (20)$$

When the link formation does not depend on the identity of individuals, then the links should be randomly distributed with respect to identity. Therefore, the index measures the difference between the expected and actual number of links among individuals of different groups, as a fraction of the expected links. An index of 0 means that the actual network closely resembles one in which links are formed at random. Higher values indicate more segregation. In this paper segregation is measured using the index[26]

$$SEG = \max\{0, FSI\} \quad (21)$$

The index varies between 0 and 1, where the maximum corresponds to a network in which there are no cross-group links.

To complete the derivation of the index, the expected number of cross-group links is computed as

$$\begin{aligned} \mathbb{E}[n_{AB}] &= \frac{(n_{AA} + n_{AB})(n_{AB} + n_{BB})}{n_{AA} + n_{AB} + n_{BA} + n_{BB}} \\ \mathbb{E}[n_{BA}] &= \frac{(n_{BA} + n_{BB})(n_{AA} + n_{BA})}{n_{AA} + n_{AB} + n_{BA} + n_{BB}} \end{aligned}$$

[26]The index (20) varies between -1 and 1. However, the interpretation of the index when it assumes negative values is not clear. Therefore Freeman (1972) suggests to use only when it is nonnegative, to measure the presence of segregation

C Posterior estimates (complete tables)

Table 3: Model (2) in Table 2

	mean	median	std. dev.	5 pctile	95 pctile
A. DIRECT UTILITY (u_{ij})					
CONSTANT	-5.5381	-5.5384	0.0261	-5.5805	-5.4942
SAME GENDER	0.1850	0.1850	0.0133	0.1631	0.2069
SAME GRADE	2.2384	2.2384	0.0030	2.2334	2.2434
WHITE-WHITE	0.5414	0.5413	0.0048	0.5335	0.5494
BLACK-BLACK	0.3660	0.3661	0.0173	0.3375	0.3943
HISP-HISP	1.6794	1.6794	0.0322	1.6267	1.7324
ATTRACTIVE i (Physical)	0.3068	0.3070	0.0270	0.2623	0.3509
ATTRACTIVE j (Physical)	0.2322	0.2322	0.0076	0.2198	0.2448
ATTRACTIVE i (Personality)	0.0063	0.0061	0.0128	-0.0145	0.0275
ATTRACTIVE j (Personality)	0.8678	0.8679	0.0173	0.8391	0.8959
Income i - Income j (logs)	0.1462	0.1461	0.0068	0.1351	0.1574
Income i + Income j(logs)	-0.0806	-0.0806	0.0049	-0.0885	-0.0725
FRACTION WHITES	-0.4814	-0.4814	0.0320	-0.5338	-0.4284
FRACTION BLACKS	3.0985	3.0984	0.0156	3.0730	3.1242
FRACTION HISP	2.4440	2.4439	0.0207	2.4100	2.4781
WHITE-WHITE * FRACTION WHITES	1.0094	1.0095	0.0267	0.9659	1.0532
BLACK-BLACK * FRACTION BLACKS	0.1478	0.1478	0.0095	0.1322	0.1633
HISP-HISP * FRACTION HISP	-1.4255	-1.4258	0.0309	-1.4758	-1.3744
SCHOOL 1	-2.1181	-2.1186	0.0616	-2.2192	-2.0164
SCHOOL 2	1.5065	1.5066	0.0105	1.4891	1.5236
SCHOOL 3	-0.0532	-0.0531	0.0299	-0.1022	-0.0040
SCHOOL 4	1.1044	1.1043	0.0154	1.0793	1.1297
SCHOOL 5	1.3247	1.3246	0.0091	1.3098	1.3397
SCHOOL 6	0.4666	0.4667	0.0224	0.4300	0.5037
SCHOOL 7	2.3742	2.3741	0.0261	2.3313	2.4171
SCHOOL 8	0.2750	0.2750	0.0382	0.2125	0.3374
SCHOOL 9	-1.3631	-1.3631	0.0281	-1.4090	-1.3171
SCHOOL 10	1.7814	1.7817	0.0339	1.7255	1.8369
SCHOOL 11	-1.4060	-1.4059	0.0094	-1.4216	-1.3905
SCHOOL 12	2.9026	2.9027	0.0241	2.8627	2.9422
SCHOOL 13	0.3076	0.3076	0.0420	0.2388	0.3770
B. MUTUAL UTILITY (m_{ij})					
CONSTANT	1.1853	1.1852	0.0388	1.1218	1.2484
SAME GENDER	1.1652	1.1653	0.0121	1.1452	1.1849
SAME GRADE	-1.6882	-1.6883	0.0210	-1.7228	-1.6537
WHITE-WHITE	0.0073	0.0074	0.0132	-0.0147	0.0289
BLACK-BLACK	0.7468	0.7468	0.0318	0.6943	0.7992
HISP-HISP	0.7779	0.7778	0.0089	0.7635	0.7925
C. INDIRECT UTILITY AND POPULARITY (v_{ij})					
CONSTANT	-0.2891	-0.2890	0.0129	-0.3105	-0.2680
SAME GENDER	0.1721	0.1721	0.0053	0.1635	0.1808
SAME GRADE	-0.3145	-0.3145	0.0066	-0.3254	-0.3038
WHITE-WHITE	0.2239	0.2238	0.0085	0.2099	0.2379
BLACK-BLACK	-0.1364	-0.1364	0.0146	-0.1605	-0.1124
HISP-HISP	0.4328	0.4327	0.0105	0.4157	0.4501

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table 4: Model (1) in Table 2

	mean	median	std. dev.	5 pctile	95 pctile
CONSTANT	-6.9201	-6.9196	0.0393	-6.9854	-6.8562
SAME GENDER	-0.4545	-0.4546	0.0204	-0.4884	-0.4213
SAME GRADE	2.3124	2.3124	0.0030	2.3075	2.3174
WHITE-WHITE	0.3504	0.3505	0.0061	0.3402	0.3603
BLACK-BLACK	0.1443	0.1444	0.0183	0.1135	0.1742
HISP-HISP	1.8597	1.8598	0.0297	1.8109	1.9085
ATTRACTIVE i (Physical)	0.2757	0.2760	0.0267	0.2314	0.3191
ATTRACTIVE j (Physical)	-0.0410	-0.0411	0.0104	-0.0583	-0.0241
ATTRACTIVE i (Personality)	-0.4402	-0.4399	0.0152	-0.4657	-0.4158
ATTRACTIVE j (Personality)	1.0672	1.0672	0.0179	1.0378	1.0966
Income i - Income j (logs)	0.1793	0.1792	0.0071	0.1676	0.1911
Income i + Income j (logs)	-0.0882	-0.0882	0.0050	-0.0963	-0.0800
FRACTION WHITES	0.9070	0.9067	0.0465	0.8312	0.9843
FRACTION BLACKS	3.2238	3.2237	0.0153	3.1989	3.2491
FRACTION HISP	2.5240	2.5237	0.0211	2.4900	2.5593
WHITE-WHITE * FRACTION WHITES	1.3962	1.3959	0.0271	1.3526	1.4409
BLACK-BLACK * FRACTION BLACKS	0.4664	0.4665	0.0127	0.4457	0.4875
HISP-HISP * FRACTION HISP	-1.5643	-1.5643	0.0305	-1.6135	-1.5135
SCHOOL 1	-3.4873	-3.4871	0.0739	-3.6103	-3.3653
SCHOOL 2	1.8278	1.8278	0.0115	1.8089	1.8469
SCHOOL 3	-0.5626	-0.5623	0.0317	-0.6159	-0.5110
SCHOOL 4	0.4159	0.4159	0.0219	0.3796	0.4516
SCHOOL 5	1.4366	1.4365	0.0082	1.4232	1.4503
SCHOOL 6	1.3884	1.3882	0.0311	1.3376	1.4399
SCHOOL 7	2.8597	2.8592	0.0283	2.8139	2.9071
SCHOOL 8	1.2675	1.2672	0.0446	1.1948	1.3421
SCHOOL 9	-1.9436	-1.9431	0.0306	-1.9940	-1.8944
SCHOOL 10	1.7678	1.7679	0.0352	1.7091	1.8254
SCHOOL 11	-0.9222	-0.9224	0.0144	-0.9455	-0.8981
SCHOOL 12	3.5492	3.5491	0.0297	3.5004	3.5986
SCHOOL 13	-0.3995	-0.3996	0.0445	-0.4720	-0.3257

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table 5: Model (4) in Table 2

	mean	median	std. dev.	5 pctile	95 pctile
A. DIRECT UTILITY (u_{ij})					
CONSTANT	-5.9132	-5.9131	0.0146	-5.9373	-5.8894
MALE	0.0463	0.0464	0.0119	0.0264	0.0657
WHITE	0.0044	0.0045	0.0163	-0.0224	0.0310
BLACK	1.1310	1.1311	0.0063	1.1208	1.1414
HISP	2.2806	2.2804	0.0303	2.2308	2.3306
INCOME	-1.6492	-1.6490	0.0381	-1.7122	-1.5869
SAME GENDER	0.4851	0.4851	0.0155	0.4597	0.5107
SAME GRADE	2.0113	2.0113	0.0264	1.9674	2.0545
WHITE-WHITE	0.5720	0.5719	0.0129	0.5510	0.5933
BLACK-BLACK	1.1445	1.1442	0.0147	1.1208	1.1691
HISP-HISP	-0.2269	-0.2270	0.0172	-0.2553	-0.1986
BEAUTY i	-2.2413	-2.2411	0.0382	-2.3044	-2.1790
BEAUTY j	1.5861	1.5857	0.0207	1.5525	1.6204
PERSONALITY i	-0.1570	-0.1570	0.0100	-0.1736	-0.1404
PERSONALITY j	-0.7390	-0.7388	0.0185	-0.7698	-0.7088
Income i - Income j	0.9012	0.9010	0.0208	0.8672	0.9356
Income i + Income j	0.9244	0.9242	0.0220	0.8885	0.9607
FRACTION WHITES	-1.4420	-1.4420	0.0091	-1.4569	-1.4269
FRACTION BLACKS	1.8309	1.8309	0.0119	1.8114	1.8504
FRACTION HISP	0.7798	0.7798	0.0106	0.7624	0.7970
WHITE-WHITE * FRACTION WHITE	2.7840	2.7831	0.0504	2.7028	2.8685
BLACK-BLACK * FRACTION BLACKS	0.4028	0.4028	0.0063	0.3923	0.4130
HISP-HISP * FRACTION HISP	-1.3630	-1.3629	0.0075	-1.3754	-1.3508
SCHOOL 1	-0.0766	-0.0766	0.0279	-0.1220	-0.0307
SCHOOL 2	1.3889	1.3890	0.0273	1.3436	1.4337
SCHOOL 3	1.8308	1.8306	0.0185	1.8005	1.8610
SCHOOL 4	1.4277	1.4276	0.0173	1.3996	1.4565
SCHOOL 5	1.9201	1.9201	0.0145	1.8961	1.9440
SCHOOL 6	-0.7518	-0.7519	0.0197	-0.7841	-0.7191
SCHOOL 7	0.0355	0.0355	0.0135	0.0129	0.0576
SCHOOL 8	-0.5121	-0.5122	0.0228	-0.5494	-0.4746
SCHOOL 9	-2.6615	-2.6613	0.0559	-2.7538	-2.5701
SCHOOL 10	1.1371	1.1374	0.0345	1.0796	1.1937
SCHOOL 11	-0.8724	-0.8725	0.0274	-0.9173	-0.8271
SCHOOL 12	1.6418	1.6419	0.0207	1.6078	1.6758
SCHOOL 13	1.3257	1.3248	0.0526	1.2412	1.4140
B. MUTUAL UTILITY (m_{ij})					
CONSTANT	6.1668	6.1659	0.0408	6.1010	6.2346
SAME GENDER	1.0716	1.0716	0.0153	1.0462	1.0967
SAME GRADE	-3.0514	-3.0510	0.0220	-3.0882	-3.0160
WHITE-WHITE	-0.6017	-0.6016	0.0186	-0.6322	-0.5711
BLACK-BLACK	1.1177	1.1175	0.0261	1.0750	1.1613
HISP-HISP	-1.4659	-1.4655	0.0229	-1.5033	-1.4287
C. INDIRECT UTILITY AND POPULARITY (v_{ij})					
CONSTANT	-0.4705	-0.4705	0.0071	-0.4823	-0.4587
SAME GENDER	-0.4074	-0.4072	0.0069	-0.4188	-0.3962
SAME GRADE	0.1136	0.1136	0.0095	0.0981	0.1293
WHITE-WHITE	0.1856	0.1857	0.0090	0.1708	0.2004
BLACK-BLACK	0.1372	0.1371	0.0081	0.1239	0.1507
HISP-HISP	-0.5067	-0.5066	0.0111	-0.5249	-0.4886

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table 6: Model (3) in Table 2

	mean	median	std. dev.	5 pctile	95 pctile
CONSTANT	-6.6500	-6.6499	0.0189	-6.6812	-6.6191
MALE	-0.1517	-0.1516	0.0105	-0.1690	-0.1349
WHITE	-0.1710	-0.1709	0.0139	-0.1938	-0.1484
BLACK	1.0451	1.0451	0.0048	1.0371	1.0530
HISP	2.0990	2.0991	0.0235	2.0604	2.1375
INCOME	-2.0543	-2.0542	0.0319	-2.1075	-2.0025
SAME GENDER	0.2067	0.2068	0.0131	0.1848	0.2281
SAME GRADE	2.3817	2.3814	0.0211	2.3469	2.4166
WHITE-WHITE	1.0138	1.0136	0.0133	0.9921	1.0358
BLACK-BLACK	1.6491	1.6489	0.0159	1.6233	1.6754
HISP-HISP	0.3186	0.3184	0.0166	0.2914	0.3463
ATTRACTIVE i (Physical)	-2.3568	-2.3567	0.0296	-2.4057	-2.3084
ATTRACTIVE j (Physical)	2.5166	2.5163	0.0255	2.4750	2.5590
ATTRACTIVE i (Personality)	-0.4964	-0.4964	0.0087	-0.5108	-0.4821
ATTRACTIVE j (Personality)	-1.0932	-1.0930	0.0165	-1.1205	-1.0664
Income i - Income j (logs)	0.8883	0.8883	0.0141	0.8654	0.9116
Income i + Income j (logs)	1.0947	1.0947	0.0177	1.0660	1.1242
FRACTION WHITES	-1.7088	-1.7087	0.0074	-1.7210	-1.6966
FRACTION BLACKS	1.3416	1.3419	0.0128	1.3205	1.3625
FRACTION HISP	0.8397	0.8397	0.0084	0.8260	0.8535
WHITE-WHITE * FRACTION WHITES	4.3915	4.3908	0.0526	4.3059	4.4785
BLACK-BLACK * FRACTION BLACKS	0.2528	0.2529	0.0061	0.2428	0.2627
HISP-HISP * FRACTION HISP	-1.6908	-1.6907	0.0088	-1.7053	-1.6766
SCHOOL 1	-0.2439	-0.2439	0.0222	-0.2807	-0.2075
SCHOOL 2	1.7809	1.7807	0.0217	1.7450	1.8169
SCHOOL 3	1.7858	1.7858	0.0146	1.7616	1.8097
SCHOOL 4	1.9064	1.9061	0.0176	1.8780	1.9355
SCHOOL 5	2.2429	2.2428	0.0127	2.2221	2.2642
SCHOOL 6	-1.4227	-1.4226	0.0179	-1.4523	-1.3933
SCHOOL 7	-0.2224	-0.2223	0.0124	-0.2428	-0.2024
SCHOOL 8	0.2460	0.2457	0.0232	0.2084	0.2842
SCHOOL 9	-2.7969	-2.7967	0.0425	-2.8673	-2.7275
SCHOOL 10	0.8911	0.8914	0.0262	0.8480	0.9341
SCHOOL 11	-1.0609	-1.0608	0.0206	-1.0949	-1.0270
SCHOOL 12	0.9857	0.9859	0.0206	0.9516	1.0191
SCHOOL 13	2.9091	2.9085	0.0523	2.8237	2.9954

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table 7: Model (6) in Table 2

	mean	median	std. dev.	5 pctile	95 pctile
A. DIRECT UTILITY (u_{ij})					
CONSTANT	-5.8070	-5.8070	0.0068	-5.8185	-5.7958
MALE	0.2350	0.2348	0.0081	0.2212	0.2483
WHITE	0.3023	0.3023	0.0120	0.2826	0.3219
BLACK	1.1801	1.1801	0.0028	1.1755	1.1847
HISP	2.0295	2.0297	0.0197	1.9967	2.0616
INCOME	-1.4645	-1.4645	0.0259	-1.5072	-1.4216
SAME GENDER	0.7644	0.7644	0.0093	0.7491	0.7797
SAME GRADE	2.1800	2.1798	0.0162	2.1533	2.2067
WHITE-WHITE	0.2739	0.2739	0.0063	0.2637	0.2839
BLACK-BLACK	0.9405	0.9405	0.0098	0.9246	0.9570
HISP-HISP	-0.1394	-0.1395	0.0098	-0.1554	-0.1226
BEAUTY i	-1.9430	-1.9432	0.0258	-1.9855	-1.9001
BEAUTY j	1.2609	1.2609	0.0121	1.2412	1.2812
PERSONALITY i	-0.1631	-0.1633	0.0057	-0.1725	-0.1533
PERSONALITY j	-0.3939	-0.3939	0.0135	-0.4164	-0.3716
Income i - Income j	0.7403	0.7401	0.0134	0.7178	0.7621
Income i + Income j	0.6892	0.6894	0.0149	0.6643	0.7136
FRACTION WHITES	-1.6126	-1.6124	0.0060	-1.6228	-1.6029
FRACTION BLACKS	1.9618	1.9618	0.0063	1.9514	1.9722
FRACTION HISP	0.7731	0.7731	0.0066	0.7623	0.7839
WHITE-WHITE * FRACTION WHITE	2.3272	2.3271	0.0340	2.2719	2.3837
BLACK-BLACK * FRACTION BLACKS	0.2516	0.2515	0.0066	0.2410	0.2624
HISP-HISP * FRACTION HISP	-1.1400	-1.1399	0.0059	-1.1494	-1.1302
SCHOOL 1	-0.1335	-0.1336	0.0163	-0.1603	-0.1067
SCHOOL 2	1.4996	1.4994	0.0169	1.4722	1.5276
SCHOOL 3	1.8785	1.8785	0.0102	1.8620	1.8954
SCHOOL 4	1.3724	1.3724	0.0106	1.3553	1.3898
SCHOOL 5	1.6828	1.6827	0.0088	1.6686	1.6973
SCHOOL 6	-1.0683	-1.0679	0.0128	-1.0902	-1.0481
SCHOOL 7	-0.9817	-0.9815	0.0280	-1.0285	-0.9346
SCHOOL 8	-0.5932	-0.5929	0.0203	-0.6274	-0.5602
SCHOOL 9	0.2444	0.2442	0.0109	0.2267	0.2624
SCHOOL 10	-1.1949	-1.1948	0.0168	-1.2230	-1.1679
SCHOOL 11	-2.3824	-2.3821	0.0379	-2.4446	-2.3196
SCHOOL 12	1.2316	1.2318	0.0248	1.1911	1.2720
SCHOOL 13	-1.4722	-1.4719	0.0203	-1.5061	-1.4389
SCHOOL 14	1.8479	1.8480	0.0084	1.8339	1.8617
SCHOOL 15	0.5666	0.5663	0.0301	0.5176	0.6166
B. MUTUAL UTILITY (m_{ij})					
CONSTANTm	5.3139	5.3137	0.0257	5.2721	5.3572
SAME GENDERm	1.1539	1.1536	0.0088	1.1397	1.1688
SAME GRADEm	-3.0575	-3.0575	0.0158	-3.0831	-3.0317
WHITE-WHITEm	-0.4960	-0.4959	0.0120	-0.5162	-0.4766
BLACK-BLACKm	0.7067	0.7068	0.0178	0.6771	0.7362
HISP-HISPm	-1.4639	-1.4639	0.0120	-1.4839	-1.4442
C. INDIRECT UTILITY AND POPULARITY (v_{ij})					
CONSTANTv	-0.4308	-0.4309	0.0048	-0.4386	-0.4230
SAME GENDERv	-0.3987	-0.3987	0.0045	-0.4061	-0.3914
SAME GRADEv	0.3266	0.3266	0.0072	0.3148	0.3384
WHITE-WHITEv	0.2978	0.2978	0.0042	0.2909	0.3047
BLACK-BLACKv	0.1202	0.1203	0.0088	0.1057	0.1343
HISP-HISPv	-0.2859	-0.2860	0.0059	-0.2958	-0.2759

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 20000 parameter simulations, and 10000 network simulations for each parameter proposal.

Table 8: Model (5) in Table 2

	mean	median	std. dev.	5 pctile	95 pctile
A. DIRECT UTILITY (u_{ij})					
CONSTANT	-7.2182	-7.2151	0.0329	-7.2761	-7.1685
MALE	-0.2718	-0.2717	0.0301	-0.3208	-0.2232
WHITE	0.0440	0.0445	0.0455	-0.0286	0.1136
BLACK	0.7074	0.7049	0.0138	0.6880	0.7323
HISP	1.4590	1.4588	0.0213	1.4250	1.4946
INCOME	-1.8738	-1.8740	0.0279	-1.9215	-1.8279
SAME GENDER	0.3154	0.3153	0.0156	0.2898	0.3406
SAME GRADE	2.5185	2.5173	0.0297	2.4713	2.5689
WHITE-WHITE	0.9959	0.9832	0.0534	0.9271	1.0975
BLACK-BLACK	1.5347	1.5251	0.0437	1.4755	1.6159
HISP-HISP	0.7130	0.7030	0.0530	0.6427	0.8099
BEAUTY i	-1.9291	-1.9295	0.0266	-1.9732	-1.8841
BEAUTY j	2.7615	2.7616	0.0242	2.7218	2.8005
PERSONALITY i	-0.8646	-0.8571	0.0401	-0.9359	-0.8087
PERSONALITY j	-0.6361	-0.6332	0.0238	-0.6817	-0.6017
Income i - Income j	0.9938	0.9943	0.0141	0.9695	1.0169
Income i + Income j	0.8977	0.8979	0.0164	0.8704	0.9243
FRACTION WHITES	-1.5748	-1.5614	0.0661	-1.6958	-1.4910
FRACTION BLACKS	0.7645	0.7742	0.0534	0.6684	0.8375
FRACTION HISP	1.0078	1.0023	0.0319	0.9660	1.0638
WHITE-WHITE * FRACTION WHITE	4.7269	4.7281	0.0509	4.6417	4.8081
BLACK-BLACK * FRACTION BLACKS	0.1172	0.1171	0.0125	0.0974	0.1382
HISP-HISP * FRACTION HISP	-1.3872	-1.3915	0.0297	-1.4288	-1.3364
SCHOOL 1	-0.4403	-0.4408	0.0232	-0.4783	-0.4007
SCHOOL 2	2.4641	2.4648	0.0204	2.4303	2.4969
SCHOOL 3	1.3139	1.3041	0.0418	1.2578	1.3919
SCHOOL 4	2.4282	2.4233	0.0356	2.3778	2.4915
SCHOOL 5	2.8177	2.8181	0.0191	2.7867	2.8487
SCHOOL 6	-1.7375	-1.7362	0.0249	-1.7802	-1.6982
SCHOOL 7	-0.7972	-0.7982	0.0387	-0.8597	-0.7306
SCHOOL 8	-1.6076	-1.6071	0.0584	-1.7031	-1.5125
SCHOOL 9	0.4031	0.4053	0.0193	0.3675	0.4317
SCHOOL 10	-0.9558	-0.9558	0.0405	-1.0210	-0.8891
SCHOOL 11	-2.5207	-2.5209	0.0388	-2.5865	-2.4563
SCHOOL 12	0.7806	0.7750	0.0429	0.7171	0.8594
SCHOOL 13	-1.4684	-1.4657	0.0440	-1.5433	-1.3997
SCHOOL 14	0.0815	0.0818	0.0257	0.0388	0.1237
SCHOOL 15	3.6894	3.6896	0.0506	3.6055	3.7699

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 20000 parameter simulations, and 10000 network simulations for each parameter proposal.