C Unobserved heterogeneity

It is possible to incorporate unobserved heterogeneity or random coefficients in the model. However this would significantly increase the computational cost of estimation. The simplest way to introduce unobserved heterogeneity is to model the preference shock ε_{ij} as incorporating individual random effects. The decision of the player to form a link is modified as follows

$$U_i\left(g_{ij}=1, g_{-ij}, X\right)+\eta_i+\eta_j+\nu_{ij1} \geq U_i\left(g_{ij}=0, g_{-ij}, X\right)+\eta_i+\nu_{ij0} \tag{41}$$

where ν_{ij} is an i.i.d. shock with logistic distribution and the vector $\eta=\{\eta_1, \ldots, \eta_n\}$ is drawn at time 0 from a known distribution $W(\eta)$. In this formulation, we assume that the players observe the random effect η but the econometrician does not. Notice that the random effect of player i cancels out, while the choice of linking j is conditional on the random effect of player j (which is present only when the link is formed).

Conditioning on the realization of the vector $\eta \in \Upsilon$, the potential function is modified as follows

$$\mathcal{Q}\left(g, X, \theta ; \eta\right)=Q\left(g, X, \theta\right)+\sum_{i=1}^n \sum_{j=1}^n g_{ij} \eta_j \tag{42}$$

To compute the unconditional likelihood we need to integrate out the unobserved vector η to obtain

$$\pi\left(g, X, \theta\right)=\int_{\Upsilon} \frac{\exp \left[\mathcal{Q}\left(g, X, \theta ; \eta\right)\right]}{\sum_{\omega \in \mathcal{G}} \exp \left[\mathcal{Q}\left(\omega, X, \theta ; \eta\right)\right]} dW\left(\eta\right) \tag{43}$$

The integral above can be computed using Monte Carlo techniques, as it is standard in the empirical industrial organization literature or labor economics. However, the model does not allow standard Monte Carlo, because of the normalizing constant.

A more feasible strategy is to use data augmentation and Markov Chain Monte Carlo methods as in the discrete choice literature (Rossi et al. (1996), Athey and Imbens (2007)). Conditioning on the realization of the unobserved component η, we can use the exchange algorithm to sample from the posterior distribution of θ. Conditioning on the proposed θ we can use a Metropolis-Hastings step to sample the unobserved component η.

Given an initial (θ, η) at simulation s, we propose a new θ' and use the exchange algorithm to accept or reject the proposal. Given the new value of θ_{s+1}, we propose a new vector of unobserved components η' and accept using a Metropolis-Hastings step. The probability of η, conditioning on (θ, g, X) is

$$\Pr\left(\eta | g, X, \theta\right)=\frac{W\left(\eta\right) \pi\left(g, X, \theta ; \eta\right)}{\pi\left(g, X, \theta\right)} \tag{44}$$

The Metropolis-Hastings step proceeds by proposing a new η' from a distribution $q_\eta\left(\eta' | \eta\right)$, which is accepted with probability

$$\alpha_\eta\left(\eta, \eta', g, \theta_s\right)=\left\{1, \frac{W\left(\eta'\right) \pi\left(g, X, \theta ; \eta'\right) q_\eta\left(\eta | \eta'\right)}{W\left(\eta\right) \pi\left(g, X, \theta ; \eta\right) q_\eta\left(\eta' | \eta\right)}\right\} \tag{45}$$

Similar ideas apply to random coefficients. However, as discussed in Graham (2014), when we observe only one network in the data, it is not possible to separately identify the linking externalities and the unobserved heterogeneity.

The main cost of these extensions is the increased computational burden, which may be substantial.